11.02 Nuclear radiation (1)

Isotope essentials

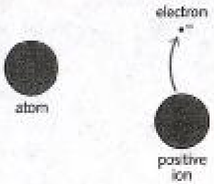
Different versions of the same element are called isotopes. Their atoms have different numbers of neutrons in the nucleus.

For example, lithium is a mixture of two isotopes: lithium-6 (with 3 protons and 3 neutrons in the nucleus) and lithium-7 (with 3 protons and 4 neutrons).

Some materials contain atoms with unstable nuclei. In time, each unstable nucleus disintegrates (breaks up). As it does so, it shoots out a tiny particle and, in some cases, a burst of wave energy as well. The particles and waves 'radiate' from the nucleus, so they are somtimes called **nuclear radiation**. Materials which emit nuclear radiation are known as **radioactive** materials. The disintegration of a nucleus is called **radioactive decay**.

Some of the materials in nuclear power stations are highly radioactive. But nuclear radiation comes from natural sources as well. Although it is convenient to talk about 'radicactive materials', it is really particular isotopes of an element that are radioactive. Here are some examples:

isotopes		
stable nuclei	*unstable nuclei, radioactive*	*found in*
carbon-12 carbon-13	carbon-14	air, plants, animals
potassium-39	potassium-40	rocks, plants, sea water
	uranium-234 uranium-235 uranium-238	rocks



If an atom loses (or gains) an electron, it becomes an ion.

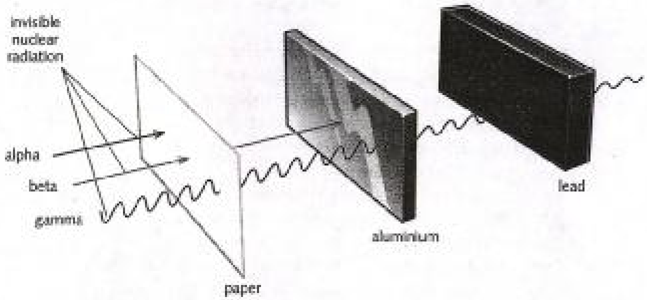
Ionizing radiation

Ions are charged atoms (or groups of atoms). Atoms become ions when they lose (or gain) electrons. Nuclear radiation can remove electrons from atoms in its path, so it has an **ionizing** effect. Other forms of ionizing radiation include ultraviolet and X-rays.

If a gas becomes ionized, it will conduct an electric current. In living things, ionization can damage or destroy cells (see the next spread).

Alpha, beta, and gamma radiation

There are three main types of nuclear radiation: **alpha particles**, **beta particles**, and **gamma rays**. Gamma rays are the most penetrating and alpha particles the least, as shown below:

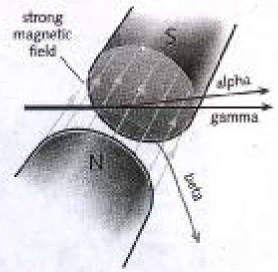


Discovering radioactivity

Henri Becquerel discovered radioactivity, by accident, in 1896. When he left some uranium salts next to a wrapped photographic plate, he found that the plate had become 'fogged', and realized that some invisible, penetrating radiation must be coming from the uranium.

type of radiation	alpha particles (α)	beta particles (β)	gamma rays (γ)
	 each particle is 2 protons + 2 neutrons (it is identical to a nucleus of helium-4)	 each particle is an electron (created when the nucleus decays)	 electromagnetic waves similar to X-rays
relative charge compared with charge on proton	+2	−1	0
mass	high, compared with betas	low	–
speed	up to 0.1 × speed of light	up to 0.9 × speed of light	speed of light
ionizing effect	strong	weak	very weak
penetrating effect	not very penetrating: stopped by a thick sheet of paper, or by skin, or by a few centimetres of air	penetrating, but stopped by a few millimetres of aluminium or other metal	very penetrating: never completely stopped, though lead and thick concrete will reduce intensity
effects of fields	deflected by magnetic and electric fields	deflected by magnetic and electric fields	not deflected by magnetic or electric fields

The nature and main properties of the three types of radiation are given in the chart above. The diagram on the right shows how the different types are affected by a magnetic field. The alpha beam is a flow of positively (+) charged particles, so it is equivalent to an electric current. It is deflected in a direction given by Fleming's left-hand rule – the rule used for working out the direction of the force on a current-carrying wire in a magnetic field. The beta particles are much lighter than the alpha particles and have a negative (-) charge, so they are deflected more, and in the opposite direction. Being uncharged, the gamma rays are not deflected by the field.



Alpha and beta particles are also affected by an electric field – in other words, there is a force on them if they pass between oppositely charged plates.

Q

1 Name a radioactive isotope which occurs naturally in living things.

2 *alpha beta gamma*
Which of these three types of radiation
a) is a form of electromagnetic radiation
b) carries positive charge
c) is made up of electrons
d) travels at the speed of light
e) is the most ionizing
f) can penetrate a thick sheet of lead
g) is stopped by skin or thick paper
h) has the same properties as X-rays
i) is not deflected by an electric or magnetic field?

3 What is the difference between the atoms of an isotope that is radioactive and the atoms of an isotope that is not?

4 How is an ionized material different from one that is not ionized?

Related topics: electromagnetic waves 7.10–7.11; X-rays 7.11 and 10.10; ions 8.09; Fleming's left-hand rule 9.05; isotopes 11.01

11.03 Nuclear radiation (2)

▲ Where background radiation comes from (average proportions).

Radiation dangers

Nuclear radiation can damage or destroy living cells, and stop organs in the body working properly. It can also upset the chemical instructions in cells so that these grow abnormally and cause cancer. The greater the intensity of the radiation, and the longer the exposure time, the greater the risk.

Radioactive gas and dust are especially dangerous because they can be taken into the body with air, food, or drink. Once absorbed, they are difficult to remove, and their radiation can cause damage in cells deep in the body. Alpha radiation is the most harmful because it is the most highly ionizing.

Normally, there is much less risk from radioactive sources *outside* the body. Sources in nuclear power stations and laboratories are well shielded, and the intensity of the radiation decreases as you move away from the source. Beta and gamma rays are potentially the most harmful because they can penetrate to internal organs. Alpha particles are stopped by the skin.

Background radiation

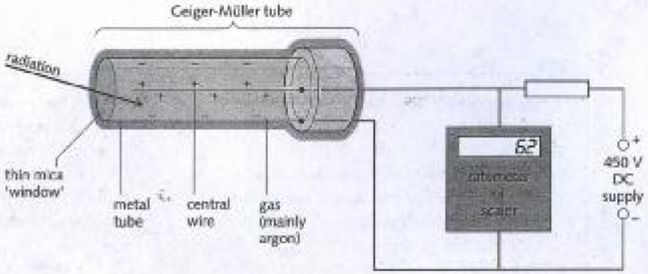
There is a small amount of radiation around us all the time because of radioactive materials in the environment. This is called **background radiation**. It mainly comes from natural sources such as soil, rocks, air, building materials, food and drink – and even space.

In some areas, over a half of the background radiation comes from radioactive radon gas (radon-222) seeping out of rocks – especially some types of granite. In high risk areas, houses may need extra underfloor ventilation to stop the gas collecting or, ideally, a sealed floor to stop it entering in the first place.

▲ This nuclear laboratory worker is about to use a GM tube and ratemeter to check for any traces of radioactive dust on her clothing.

Geiger-Müller (GM) tube

This can be used to detect alpha, beta, and gamma radiation. Its structure is shown below. The 'window' at the end is thin enough for alpha particles to pass through. If an alpha particle enters the tube, it ionizes the gas inside. This sets off a high-voltage spark across the gas and a pulse of current in the circuit. A beta particle or burst of gamma radiation has the same effect.



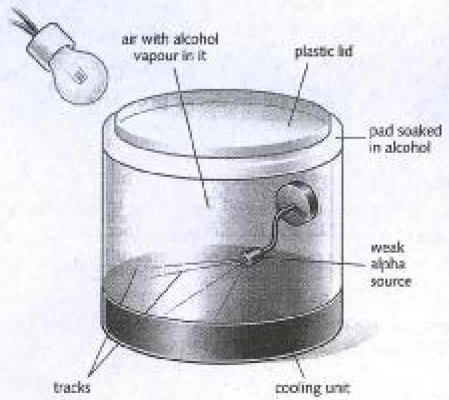
The GM tube can be connected to the following:

- **A ratemeter** This gives a reading in counts per second. For example, if 50 alpha particles were detected by the GM tube every second, the ratemeter would read 50 counts per second.
- **A scaler** This counts the *total* number of particles (or bursts of gamma radiation) detected by the tube.
- **An amplifier and loudspeaker** The loudspeaker makes a 'click' when each particle or burst of gamma radiation is detected.

When the radiation from a radioactive source is measured, the reading always *includes* any background radiation present. So an average reading for the background radiation alone must also be found and subtracted from the total.

Cloud chamber

This is useful for studying alpha particles because it makes their tracks visible. The chamber has cold alcohol vapour in the air inside it. The alpha particles make the vapour condense, so you see a trail of tiny droplets where each particle passes through. At one time, cloud chambers were widely used in nuclear research, but they have since been replaced by other devices.



Cloud chamber

Tracks of alpha particles in a cloud chamber. The colours are false and have been added to the picture. The green and yellow lines are the alpha tracks. The red line is the track of a nitrogen nucleus that has been hit by an alpha particle.

Q

1. What, on average, is the biggest single source of background radiation?
2. Radon gas seeps out of rocks underground. Why is it important to stop radon collecting in houses?
3. Which is the most dangerous type of radiation
   a) from radioactive sources outside the body
   b) from radioactive materials absorbed by the body?
4. In the experiment on the right:
   a) What is the count rate due to background radiation?
   b) What is the count rate due to the source alone?
   c) If the source emits one type of radiation only, what type is it? Give a reason for your answer.

count rate (average)...	counts per second
...with the source in place	28
...with the source and block in place	18
...with the source and block removed	2

Related topics: properties of alpha, beta, and gamma radiation 11.02

OBJECTIVES

- the Copenhagen interpretation
- wave function
- measurement problem
- Schrödinger's cat

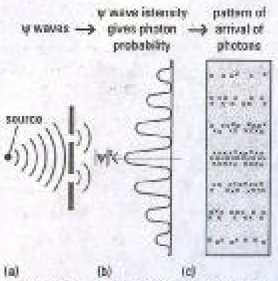
Niels Bohr is generally regarded as the architect of the Copenhagen interpretation of quantum theory. His arguments with Einstein about the new theory are physics legends. Recent experiments reinforce Bohr's point of view and the strangeness of quantum theory.

The Copenhagen interpretation

- The most complete description of matter or radiation is given by the wave function ψ.
- The wave function interacts with apparatus like a continuous classical wave.
- The probability of finding a photon (or electron, etc.) in a small volume of space is proportional to the square of the amplitude of the wave function at that point and the volume of the space concerned:

probability that the photon is found in a volume $\delta V = |\psi|^2\ \delta V$

- Photons, electrons, etc. are always observed as complete quanta.
- When an observation is made the wave function 'collapses' instantaneously everywhere.



(a) ψ waves leave the source and interact with the double slits (b) interference leads to a pattern of maxima and minima of $|\psi|^2$ at the screen (c) light arrives at the screen as photons. The distribution of photons is governed by $|\psi|^2$

Using wave–particle duality.

Wave–particle duality

Thomas Young used the double-slit interference pattern to measure the wavelength of light for the first time. However, the photoelectric effect could not be explained by wave theory, and needed Einstein's photon hypothesis, a particle theory. So what *is* light? It cannot be *just* a stream of particles and yet it cannot be a continuous wave either. This dilemma is sometimes described as **wave–particle duality**, and is not restricted to light. De Broglie predicted that matter also suffers from it. This was soon confirmed by electron diffraction and, more recently, by the diffraction and interference of whole atoms.

Wave–particle duality means that neither the particle nor the wave model can be used consistently to explain everything that light or matter does. The best model we have is that of quantum mechanics, an abstract mathematical theory which does not lend itself to any simple pictures of what is going on. This becomes obvious if you try to explain the double-slit interference pattern using photon theory. In the end, most physicists settled for the **Copenhagen interpretation**, which really abandons any hope of a 'physical picture'. It is essentially a way of relating the readings obtained on detectors (geiger counters, photographic plates, etc.) to the mathematical model of quantum theory. Not all physicists agree with this interpretation, so it's still the subject of many late-night arguments!

Some sort of interpretation

One of Einstein's most famous sayings is, 'God does not play dice'. He was referring to the so-called Copenhagen interpretation, with which he did not agree. Surprisingly though, the central idea of the Copenhagen interpretation, that the waves are linked to probability, was first suggested by Einstein himself in a letter to his friend Max Born. At the time Einstein was trying to understand how light could sometimes behave like a particle and sometimes like a wave. He wondered whether the strength of the waves at each point might be linked to the probability that photons are found there.

This idea was developed by Born into a probability model for wave–particle duality. Assume that the wave-like aspect of a photon (or electron) is represented by a **wave function**, ψ ('psi'). This behaves just like a classical wave but with one very important difference – it is not directly observable. Its *intensity* in any small volume of space is proportional to the chance that the photon is there.

This provides a sensible way to think about interference without losing the particle aspect of radiation. The wave functions interact with the apparatus, diffract through apertures, and form interference patterns, but the intensities of these patterns determine where the photons are likely to be found. So even if just one photon interacts with double slits in Young's experiment its wave function forms an interference pattern and the photon is more likely to hit the screen near a maximum (high intensity and therefore high probability) than near a minimum.

In the probability model the distribution of photons or electrons is fixed by the way the wave function interacts with the apparatus, but we only ever detect complete photons or electrons. It is a bit like tossing a coin – the probabilities of heads or tails are equal at 0.5 each, but the coin is always observed to land in one or the other state.

Collapse of the wave function

Something strange happens when a photon (or electron) is detected. Before detection the wave function represents the probability that the photon *might* be detected at a particular position. After detection it must change to represent the new state of affairs – either the photon is there (probability 1) or not (probability 0). This means the wave function changes *everywhere* whenever a measurement or observation is made. This causes problems for physicists, because it implies that a part of the wave function must respond to something that happens in a particular position far away with *no time delay* for a signal of any kind to reach it. This problem is referred to as the **quantum measurement problem** or simply the **collapse of the wave function**.

Quantum theory and the future

The nature of probability involved in quantum theory is deeper than that involved in tossing a coin. The way the coin lands could (in principle) be predicted if you knew enough about the coin and the mechanics of the toss. This is *not* the case in quantum theory. If you knew everything there was to know about a photon approaching a double slit, you could predict what would happen to its wave function, but not where among the maxima and minima the photon would end up. In classical physics unique initial conditions lead to unique outcomes – this is called a **deterministic theory**. In quantum theory the same initial conditions can lead to many different outcomes, so the future is not uniquely determined by the past. This is another radical change of thinking brought about by the new theory.

Schrödinger's cat

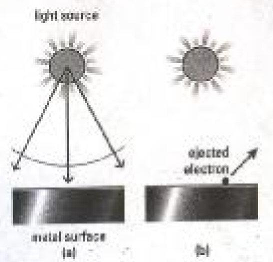
Erwin Schrödinger was responsible for introducing the idea of the wave function into physics, but he was so shocked by the implications of quantum theory that it was claimed he wished he had never had anything to do with it! To show how strange it is he invented a very famous thought experiment. If quantum theory applies to atoms it must also apply to cats and people since we are made of atoms. He imagined a cat locked in a box with a capsule of poison and a radioactive atom. If the atom decays it triggers a detector that smashes the capsule and poisons the cat. Before the box is opened there is no way of telling what has actually happened.

In the classical world the cat is either dead or alive, but in the quantum world the unobserved atom is in a **superposition of states** represented by wave functions for 'decay' and 'no decay'. If the cat too is described by a wave function, it is in a superposition of dead and alive states. It only becomes actually dead or alive when we open the box and look in. It is as if two possible realities co-exist in our universe until an *act of observation* causes the wave function to collapse and one or other state to become real! Strange though this sounds, it represents the conventional interpretation of quantum theory used by most physicists.

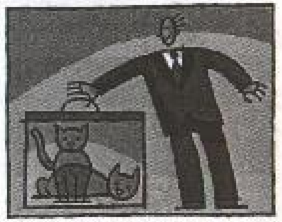
Alternative interpretations

If you don't like the Copenhagen interpretation there are alternatives. Two of the most popular are outlined below.

- **The many worlds theory** Imagine an experiment has two alternative outcomes. According to this theory both occur, but in parallel universes. Surprisingly this idea was made into a consistent model of quantum theory by Hugh Everett III and is particularly popular among quantum cosmologists.
- **The sum over histories** Another way to explain quantum theory is to assume that everything that can happen does, but in a world of potentialities. The actualities (that is, what actually does happen) are like weighted averages of all these possible histories. For example, in the double-slit experiment, the interference pattern can be derived by superposing all possible routes the photons could take from the source to the screen, including the crazy ones (like stopping halfway and taking a detour via New York). In the end the effects of the crazy ones cancel out, leaving just what can be expected by more conventional derivations.



Collapse of the wave function. (a) shows a continuous ψ wave just before a photon hits a metal surface. In (b) the ψ wave has collapsed and all the photon energy has been transferred to a single electron at one point in the metal surface (this is the photoelectric effect).



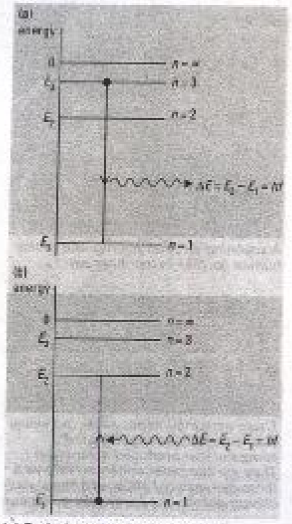
Schrödinger's cat. Just before he opens the box the quantum mechanical description of his cat has it in a superposition of live and dead states. It is only when an observation is made that the wave function collapses into one or other of these states and the fate of the cat is sealed.

Wigner's friend

Eugene Wigner took the cat paradox one step further. He suggested that, even after the box has been opened inside the laboratory, a friend of the experimenter outside the laboratory will not know the outcome of the experiment. So, for the friend, the inside of the laboratory, together with atom, capsule, cat, box, and experimenter, remains in a superposition of two states until the door is opened and its wave function collapses. Think about it – at what point in this process would the cat's fate be decided?

8.9 The quantum atom

OBJECTIVES
- Bohr–Sommerfeld atom
- electron waves in atoms
- Schrödinger atom



(a) Emission of a photon from an excited atom. (b) Absorption of a photon causes excitation.

Maths box: Bohr's quantum condition

Bohr quantized the angular momentum (mvr) of the electrons, setting it equal to an integer multiple of $h/2\pi$. This condition, added to the classical equations for electrostatic attraction and centripetal force in the Rutherford model, led to an equation for the allowed energy levels in the hydrogen atom:

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} \quad \text{where } n = 1, 2, 3, \text{etc.}$$

This equation accurately predicts the ground state and many of the excited states. Differences between levels can be used to calculate the wavelengths of hydrogen's spectral lines. These agree with observed values – e.g. in the Balmer series (see spread 8.10).

It also gives the ionization energy of hydrogen. The electron will leave the atom if its total energy is greater than or equal to zero (all bound states have negative total energy). This is equivalent to promoting an electron from the $n = 1$ to $n = \infty$ levels.

$$E_{\text{ionization}} = E_\infty - E_1 = \frac{me^4}{8\varepsilon_0^2 h^2} = 13.6\text{ eV}$$

in agreement with measured values.

Rutherford's model of the atom was useful, but had serious drawbacks. For example, Maxwell's theory of electromagnetism predicts that all accelerating charges should radiate. Electrons orbiting nuclei have a centripetal acceleration, so they should radiate. Atoms ought to collapse in a fraction of a second as the orbiting electrons radiate their energy and fall into the nucleus. On a larger scale this **synchrotron radiation** is a major power loss from circular accelerators, so it should not be dismissed lightly. How then can atoms exist?

Niels Bohr was aware of this problem. He was also aware that quantization of energy had solved other problems in classical physics, so he quantized the atom. In Rutherford's theory an electron can orbit with any energy and any radius, so that the atom has a continuous range of allowed energies. Bohr added a quantum condition (see below left) which meant the electrons could only orbit at certain radii and have certain energies. Atomic collapse was prevented because there were no allowed states between these **energy levels** and because the lowest, or **ground state**, was not at zero energy. Bohr's original theory involved circular orbits; Sommerfeld developed the theory to include the more general elliptical orbits.

Quantum jumps

If an atom is in its ground state it cannot lose energy, because there are no available states of lower energy. However, if something collides with the atom or if it absorbs a photon of sufficient energy, an electron can make a **quantum jump** to a higher allowed energy level. The atom is now in an **excited state**. Although it may remain in an excited state for some time (typically 10^{-8} s), there are now lower allowed energy levels and the electron will eventually make a quantum jump back down into one of these. As it does so, it loses an amount of energy equal to the difference between the energies of the original and final states, and radiates this as a photon. The photon frequency and wavelength are therefore fixed by the size of the energy jump.

Energy levels are labelled by the principal quantum number n which can take any integer value from 1 upwards. If an electron jumps from the $n = 3$ state to the $n = 1$ state it will emit a photon of frequency f. For this quantum jump

$$\Delta E = E_3 - E_1$$

$$\text{photon energy, } hf = \Delta E = E_3 - E_1$$

so

$$f = \frac{\Delta E}{h} = \frac{E_3 - E_1}{h} \quad \text{and} \quad \lambda = \frac{c}{f} = \frac{hc}{E_3 - E_1}$$

- Notice that the larger the energy of the quantum jump, the shorter the wavelength of the radiation emitted.

Atoms of a particular element have a characteristic structure of allowed energy levels, so there are only certain energy jumps the electrons can make. This means the photons emitted from atoms should have certain well-defined wavelengths. They do – if a gas of atoms is excited by heating it or passing an electric current through it, it emits a characteristic **line spectrum**.

Electron waves in atoms

Rutherford treated electrons in atoms like tiny charged particles. De Broglie's idea of matter waves changed all this and led to a simple pictorial way of understanding why the orbits are quantized. If the electrons in atoms act like waves, they might behave like the standing waves that are formed on a string of a musical instrument when it is plucked. Only certain wavelengths 'fit' the string and reinforce, others cancel out. This leads to a set of discrete frequencies giving the fundamental and harmonics of the instrument. It turns out that the 'fundamental' and 'harmonics' of electron waves form the ground state and excited states of the atom.

For Bohr's circular orbits the condition for electron waves to reinforce as standing waves is simply that their wavelength fits the circumference of the orbit a whole number of times. The principal quantum number n is equal to the number of complete waves that fit the circumference of the orbit, and Bohr's quantum condition becomes

$$2\pi r = n\lambda$$

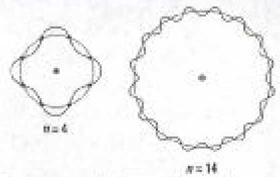
The de Broglie relation gives

$$\lambda = \frac{h}{mv}$$

so

$$2\pi r = \frac{nh}{mv} \quad \text{or} \quad mvr = \frac{nh}{2\pi}$$

which is Bohr's original condition for the quantization of electron angular momentum.



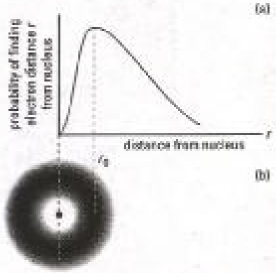
Bohr's atom. In the $n = 4$ state the circumference of the electron orbit equals 4 de Broglie wavelengths. On the right is a highly excited state ($n = 14$).

The Schrödinger atom and beyond

The piecemeal quantization of physics by Planck, Einstein, Bohr, and others was very successful, but there was no real quantum theory that could be applied to all phenomena. This was supplied by Schrödinger and Heisenberg in 1925. Their versions of quantum theory used different mathematics but were soon shown to be equivalent.

Schrödinger's version is easiest to think about. He set about finding an equation for electron wave functions like Maxwell's electromagnetic equations for light, and came up with what is now called the Schrödinger equation. He applied it to the hydrogen atom. The result was a series of three-dimensional equations for the way the electron wave function varies with position around the nucleus. The intensity of the wave function at any point (given by its magnitude squared) is proportional to the probability that the electron is in that region of space. The solutions are a discrete set of equations for the electron wave function, each one uniquely defined by three separate quantum numbers, n (corresponding to the Bohr theory), l (to do with orbital angular momentum), and m (to do with the magnetic effect of the orbit).

The 'electron clouds' that are sometimes mentioned in connection with the atom are simply a poor description of the probability distribution surrounding the nucleus. In the Schrödinger atom the electron has lost its orbit; it could be anywhere. However, the peak of the probability still corresponds to the radius of the Bohr orbit.



Schrödinger's atom. (a) Radial probability density for the 1s state of a hydrogen atom derived from the Schrödinger equation. (b) A two-dimensional representation of the 1s electron orbital. Darker shading represents greater probability and maximum probability is at $r = r_0$, equal to the Bohr radius. Higher energy orbitals are not spherical.

Chemical notation

If you have come across 1s and 2p states in chemistry, these correspond to $n = 1$, $l = 0$ and $n = 2$, $l = 1$ respectively.

Practice

1 The table gives some of the allowed energy levels for a hydrogen atom.

Principal quantum number	Energy / 10^{-18} J
1	−2.180
2	−0.545
3	−0.242
4	−0.136
5	−0.087
6	−0.061
7	−0.044
8	−0.034
9	−0.027
10	−0.022

a Draw a vertical axis to represent energy in electron volts from −15 eV (at the bottom) to 0 eV at the top. Now add a series of horizontal rungs to represent allowed energy levels for hydrogen. Label each rung with the principal quantum number and energy in electron volts.

b Show that the ionization energy for hydrogen is about 13.6 eV.

c In what part of the electromagnetic spectrum would the photon be that was emitted by a quantum jump from $n = 5$ to $n = 3$?

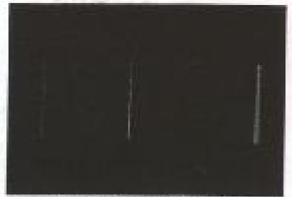
d What is the minimum frequency of light needed to eject an electron from a hydrogen atom in its $n = 4$ state?

e Show that the energy of these levels is proportional to $1/n^2$.

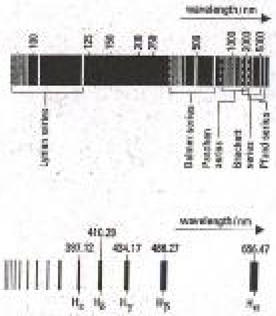
8.10

O B J E C T I V E S

- the hydrogen spectrum
- spectral series
- ionization
- band and continuous spectra



An atomic line spectrum, here the Balmer lines (see below) of the hydrogen atom.



The hydrogen spectrum. Top: the five spectral series of lines. Bottom: the Balmer series of lines in more detail.

Ionization

The first ionization energy for hydrogen is the minimum energy that must be supplied to remove an electron from the $n = 1$ level (ground state). The ground-state energy is −13.6 eV and free electrons have positive energy, so the first ionization energy for hydrogen is +13.6 eV. Another way of looking at ionization is to say that energy must be supplied to 'lift' the electron from $n = 1$ to $n = \infty$. The energy needed is then $(E_\infty - E_1)$. Since the energy approaches zero as n approaches infinity this gives the same result as the argument above.

Atomic spectra

Atomic line spectra arise from a process similar to what happens when a child jumps down stairs. When the child stands on a particular stair she has a well-defined energy which is greater than that at floor level – there are a set of discrete energy levels, one for each stair (and the child cannot stand halfway between two stairs). If she jumps from the 5th stair to the 4th she loses a fixed amount of energy that depends on the height of one step. Some of the energy she loses is given out as sound as she thumps down on the lower stair. If she jumps two or three stairs in one go she makes a bigger thump as she lands. To get back up the stairs into higher energy levels she needs an input of energy.

Electrons in atoms also have a discrete set of allowed energy levels and cannot remain in intermediate states. When an electron drops to a lower level (a quantum jump) the energy lost is released as a photon of electromagnetic radiation. The bigger the quantum jump the higher the energy (and frequency) of the emitted photon. To send it back up into a higher energy level an energy input is required – the atom must absorb a photon or gain energy in a collision. The main difference between energy levels in an atom and stairs is that the atomic energy levels are not equally spaced.

The hydrogen spectrum

The energy levels in atoms are calculated using quantum theory. The photons emitted from atoms have wavelengths fixed by the energy difference between atomic energy levels, so the first question to be asked of quantum theory was whether the energy levels it predicted could explain atomic spectra. The first atomic spectrum to be tackled was that of hydrogen, since hydrogen is the simplest atom, having just one orbital electron. (In fact it is very difficult to apply quantum theory precisely to any other atom because in all other cases the outer electrons interact with inner electrons as well as the nucleus. For this reason, most atoms are modelled using mathematical approximations.)

For hydrogen, quantum theory allows energy levels:

$$E_n = -\frac{13.6}{n^2}\ \text{eV}$$

where n is the principal quantum number and can be any positive integer.

Worked example 1

Calculate the energy of the ground state and 1st excited state of the hydrogen atom.

$$E_1 = -\frac{13.6}{1^2}\ \text{eV} = -13.6\ \text{eV} \quad \text{(ground state at } n = 1\text{)}$$

$$E_2 = -\frac{13.6}{2^2}\ \text{eV} = -3.4\ \text{eV} \quad \text{(1st excited state at } n = 2\text{)}$$

Worked example 2

Calculate the frequency and wavelength of the photon emitted when an electron makes a quantum jump from the $n = 3$ state to the ground state of the hydrogen atom.

$$E_1 = -13.6\ \text{eV} = -2.18 \times 10^{-18}\ \text{J} \quad E_3 = -\frac{13.6}{3^2}\ \text{eV} = -1.51\ \text{eV} = -2.42 \times 10^{-19}\ \text{J}$$

$$\Delta E = E_3 - E_1 = hf$$

$$f = \frac{E_3 - E_1}{h} = \frac{-2.42 \times 10^{-19}\ \text{J} - -2.18 \times 10^{-18}\ \text{J}}{6.6 \times 10^{-34}\ \text{J s}} = 2.9 \times 10^{15}\ \text{Hz}$$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^{8}\ \text{m s}^{-1}}{2.9 \times 10^{15}\ \text{Hz}} = 1.0 \times 10^{-7}\ \text{m}$$

This is in the ultraviolet part of the electromagnetic spectrum.

The spectral series

In 1885 Johann Jakob Balmer, a Swiss school teacher, noticed a simple mathematical link between the frequencies of lines in the visible part of the hydrogen spectrum.

$$f_n = k\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where k is a constant equal to 3.29×10^{15} Hz and n has values 3, 4, 5 etc.

This formula was derived directly from experimental values and neither Balmer nor anyone else at the time knew *why* it worked.

Quantum theory now shows that this series is formed by quantum jumps from excited states above $n = 2$ as the electrons fall back into the $n = 2$ level. These energy jumps correspond to photons in the visible part of the spectrum, which is why the Balmer series was the first series to be discovered. Transitions to $n = 1$ involve larger energies and result in ultraviolet photons (the Lyman series), those to $n = 3$ or above involve smaller energies and result in infrared photons (Paschen series to $n = 3$, Brackett to $n = 4$, and Pfund to $n = 5$).

A general formula for the frequencies of all hydrogen lines can be derived from the quantum formula for the energy levels by calculating the energy difference for a quantum jump from the mth to the nth level ($m > n$).

$$\Delta E = E_m - E_n = hf_{mn}$$

$$f_{mn} = \frac{E_m - E_n}{h}$$

In general the nth energy level has a value $1/n^2$ times the ground state energy.

$$E_n = \frac{E_1}{n^2} \quad \text{and} \quad E_m = \frac{E_1}{m^2}$$

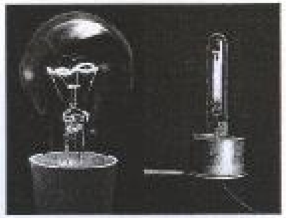
$$f_{mn} = \frac{E_1}{h}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\frac{E_1}{h} = \frac{13.6 \times 1.60 \times 10^{-19}\ \text{J}}{6.63 \times 10^{-34}\ \text{J s}} = -3.28 \times 10^{15}\ \text{Hz}$$

$$f_{mn} = 3.28 \times 10^{15}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

(The order inside the bracket changed because of the minus sign.) The Balmer series corresponds to $n = 2$.

The yellow light from sodium street lamps comes from two very similar quantum jumps in the sodium atom and consists of two very close spectral lines with wavelengths 589 nm and 589.6 nm.



A filament lamp (left), emits the characteristic continuous spectrum of a hot solid and a significant amount of its energy is in the infrared part of the spectrum. In a strip light (right), mercury vapour is excited by passing a current through it. It then emits visible and ultraviolet radiation. Sometimes the inside of the tube is coated with fluorescent material that converts the ultraviolet to visible radiation, providing an efficient source of white light.

Practice

1. Explain why energy levels are all given negative values.
2. Calculate the wavelengths of the first four spectral lines in the Lyman series. In what part of the electromagnetic spectrum would these be found?
3. Radiation from a particular quantum jump in the caesium-133 atom is used as a frequency standard to define the second. The energy levels concerned are separated by 3.8×10^{-5} eV.
   - a Calculate the frequency of this radiation.
   - b How many oscillations of this radiation occur in 1 second?
4. Fluorescent paints can absorb light at one colour and re-emit it as light of another colour.
   - a Suggest an explanation for this in terms of energy levels inside the paint atoms.
   - b Would you expect a red luminous paint to fluoresce when illuminated with green light?
   - c Would you expect green luminous paint to fluoresce when illuminated with red light?
5. Why would you normally expect a continuous spectrum from a hot solid and a line spectrum from a hot gas?
6. What is the shortest wavelength likely to be emitted from an excited hydrogen atom?

8.11 RADIOACTIVITY

OBJECTIVES
- ionizing radiation
- alpha, beta, and gamma emissions

Marie Curie won the 1903 Nobel Prize for Physics (for her work on radioactivity) and the 1911 Nobel Prize for Chemistry (for her discovery of the new elements radium and polonium). In the early 1900s it was very difficult for women to find support for a career in fundamental scientific research, and it is interesting that in 1904 Pierre Curie was given a chair at the Sorbonne in Paris whilst Marie was offered a part-time post as a physics teacher in a girls' school in Sèvres.

Lord Rutherford of Nelson.

Rutherford and his followers

No one did more to explain the mysteries of radioactivity than Ernest Rutherford, a New Zealand physicist who did most of his important work in the UK. He and his pupils discovered the nature of radioactive decay, the structure of the atom, and the constituents of the nucleus, and set the agenda for the developments that led to high-energy physics research in the rest of the twentieth century.

Radioactivity was discovered by accident in 1896. Antoine Henri Becquerel was trying to detect X-ray emission from fluorescent salts when they absorb ultraviolet light. He was inspired to do this by the discovery of X-rays a year earlier by Wilhelm Röntgen. Becquerel's method was simplicity itself. The salts (uranyl sulphate) were placed in sunlight next to a photographic plate wrapped in thick black paper. If X-rays were emitted they should penetrate the paper and darken the film. The films were darkened, and this seemed to support his hypothesis, but then he noticed something mysterious. Wrapped films kept in a drawer next to similar salts were also darkened, even though these crystals had not been exposed to sunlight. So whatever darkened the film was not caused by fluorescence – it must be an emission from something in the crystal. But what? He carried out further tests and showed that the new radiation came from uranium in the salt.

Becquerel's discovery was confirmed by the Polish physicist Marie Curie, who showed that the activity of uranium salts depended only on the amount of uranium they contained and not at all on the physical state or chemical composition of the salt. We now understand why this is – radioactive emissions come from the *nuclei* of particular atoms so they are not affected by bonding (which concerns the outer electrons) or physical conditions (like temperature, pressure, etc.).

Marie Curie made several other important discoveries in this field. She showed that thorium is also radioactive and noticed that some ores of uranium, pitchblende and chalcolite, are more active than uranium itself. She thought this must be due to new radioactive elements inside them and soon discovered radium and polonium, which are highly radioactive. Because Curie was unaware that these radiations were dangerous, she took no safety precautions as she worked with them, and her notebooks are still too radioactive to handle even today! She shared the 1903 Nobel Prize for Physics with her husband Pierre, and with Becquerel.

Some important early discoveries

- There are three types of radioactive emission, called alpha, beta, and gamma radiation. These can be separated by electric or magnetic fields.
- Radioactive emissions cause ionization, alpha radiation being most strongly ionizing and gamma radiation least strongly ionizing. Ionization is the property used to detect and measure radioactivity.
- The more strongly ionizing the radioactivity is, the more rapidly it dissipates its energy when it passes through materials. The ranges for similar energy emissions are in the order

  gamma > beta > alpha
- The activity of a source is independent of physical conditions and chemical bonding, depending only on the type of atom involved and the number of these atoms present.
- Each atom has a nucleus and radioactive decays involve nuclear transformations.
- Alpha particles are helium nuclei emitted from the nuclei of some radioactive atoms. This was shown by stopping the alpha particles inside a sealed container where they captured two electrons and became helium atoms. The presence of helium was shown by finding its spectral lines in the light emitted when an electrical discharge was passed through the tube.
- Beta particles are electrons emitted from the nuclei of some radioactive atoms (they are created in the decay and are not related to the orbital electrons in any way). This was shown by measuring their deflection in a magnetic field and showing they had the same charge-to-mass ratio (e/m) as an electron.
- Gamma rays are high-energy electromagnetic photons emitted from the nuclei of some radioactive atoms following an alpha or beta decay. These are not deflected in electric or magnetic fields. Their properties are identical to those of hard X-rays.

Nuclear transformation

Rutherford and Soddy worked out the rules for radioactive decay. All decays conserve two fundamental properties:

- charge
- number of nucleons.

Alpha decay is the emission of a helium nucleus from the nucleus of a heavy radioactive element.

A general equation for alpha decay: $^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$

An example of alpha decay: $^{238}_{92}U \to {}^{234}_{90}Th + {}^{4}_{2}\alpha$

The nucleus created in alpha decay is that of a new element two places below the original one in the periodic table. In the example above thorium is created from uranium.

Beta decay is the emission of a fast electron from the nucleus of a radioactive element.

A general equation for beta decay: $^{A}_{Z}X \to {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta$

An example of beta decay: $^{14}_{6}C \to {}^{14}_{7}N + {}^{0}_{-1}\beta$

(Actually, a more detailed description of beta decay involves the emission of an additional particle, an **anti-neutrino**.)

Notice that beta decay is more complex than alpha decay. In alpha decay some of the nucleons are hurled out of the nucleus. In beta decay a neutron inside the nucleus changes to a proton and the electron is created in the process:

$$^{1}_{0}n \to {}^{1}_{1}p + {}^{0}_{-1}e$$

This electron is the beta particle emitted. The nucleus created in beta decay is of a new element one place above the original in the periodic table.

Gamma rays are emitted when excited nuclei make quantum jumps to lower energy levels (similar to the photons emitted by quantum jumps of electrons in atoms, but of much higher energy). Often an alpha or beta decay leaves the new nucleus in an excited state, so gamma-ray emission often accompanies other radioactive decays. Gamma-ray emission does not affect the type or number of particles present in the nucleus. Sometimes excited states are represented using an asterisk.

A general equation for gamma emission: $^{A}_{Z}X^{*} \to {}^{A}_{Z}X + {}^{0}_{0}\gamma$

$^{0}_{0}\gamma$ is a photon of gamma radiation.

Definition: activity

The **activity**, A, of a radioactive source is the number of disintegrations per second in the source. The SI unit for activity is the becquerel (Bq):

1 becquerel = 1 disintegration per second

1 Bq = 1 s^{-1}

Balancing equations

The A, Z notation for nuclides has been extended to beta particles in order to emphasize the conservation laws. To be consistent the mass number A has now become **baryon number** (protons and neutrons both have B = 1 and electrons have B = 0), and the atomic number Z has now become charge Q, so that electrons (to which atomic numbers do not apply) can be recorded as Z = –1 because of their single quantum of negative charge.

$$^{1}_{0}n \to {}^{1}_{1}p + {}^{0}_{-1}e$$

B 1 = 1 + 0

Q 0 = 1 – 1

Smoke detectors contain a small amount of americium-241, an alpha emitter. When smoke (or other invisible products of combustion) enters the detector it reduces the number of alpha particles reaching an electronic detector. This triggers an alarm.

PRACTICE

1 Use your understanding of energy levels and quantum jumps in atoms to explain why you would not expect X-rays to be emitted from fluorescent salts when they have been irradiated by ultraviolet light.

2 Potassium-40 ($^{40}_{19}K$) decays by beta emission to calcium.

a Write down a transformation equation for this decay.

b What quantities are conserved in the decay? Show how they are conserved.

c How is it possible for one element to change into another in a radioactive decay?

d Explain where the electron emitted in the decay came from.

e Suggest a reason why some nuclei are prone to beta decay.

3 Plutonium-239 ($^{239}_{94}Pu$) decays by alpha emission to an isotope of uranium.

a How many protons and neutrons are present in the uranium isotope?

b Why are alpha particles so strongly ionizing?

c Write down a transformation equation for this decay.

8.13

OBJECTIVES

- the decay equation
- the decay constant
- half-life
- activity

Decay constant and probability

The equation

$$\frac{\delta N}{\delta t} = -\lambda N$$

can be rearranged to give an expression for the decay constant λ:

$$\lambda = \frac{1}{\delta t}\left(\frac{-\delta N}{N}\right)$$

The bracketed term is the fractional change in N during a time δt. This is equal to the chance of decay in that time, so λ is equal to the probability of decay per unit time in the limit of short time intervals.

THE DECAY EQUATION

The mathematical analysis of radioactive decay is based on two simple assumptions:

- Decay is completely random.
- The rate of decay is directly proportional to the number of unstable nuclei present.

Maths box: the decay equation

The second assumption above can be written down as an equation by considering the change in number of nuclei δN in a short time δt.

rate of change of number of nuclei = $\delta N/\delta t$

This is proportional to the number of nuclei present:

$$\frac{\delta N}{\delta t} \propto -N$$

The constant of proportionality is called the decay constant λ.
The negative sign shows that N is decreasing.

$$\frac{\delta N}{\delta t} = -\lambda N$$

If the time interval δt is made smaller and smaller ($\delta t \to 0$) the equation becomes:

$$\frac{dN}{dt} = -\lambda N$$

This final equation shows that the rate of change of N is proportional to N itself, a recipe for exponential decay. It is a first-order differential equation whose solution can be written as:

$$N = N_0 e^{-\lambda t}$$

where N_0 is the original number of unstable nuclei present at $t = 0$.

The decay equation

The number of nuclei N that remain at time t from an initial population of N_0 unstable nuclei is given by:

$$N = N_0 e^{-\lambda t}$$

where λ is the decay constant for the nuclide concerned.

The term $e^{-\lambda t}$ can be interpreted as the fraction remaining after time t: this fraction decays exponentially with a constant half-life, $t_{\frac{1}{2}}$.

It is worth pointing out that this equation was derived by assuming that radioactive decays occur completely at random. The fact that the exponential decay equation works confirms the idea that there is no law of nature that determines when a particular nucleus decays.

The activity of a source

Activity is defined as the number of disintegrations per second taking place inside the source, and is measured in becquerels (Bq).

1 Bq = 1 disintegration per second

There is an older unit which is still often used for activity: the curie (Ci).

1 Ci = 3.7×10^{10} disintegrations per second = 3.7×10^{10} Bq

School sources have a total activity which is typically 5.0 μCi = 1.9×10^5 Bq.

Activity A (a positive number) is equal to the number of decays per second, so it is simply minus the rate of change of N:

$$A = -\frac{dN}{dt} \quad \text{or} \quad A = \lambda N$$

In most experiments in radioactivity you will measure activity, not N. The fact that the activity is proportional to N means that it will fall off exponentially with exactly the same half-life as N does. So measuring the half-life of the activity is equivalent to measuring the half-life of the source.

Half-life and decay constant

The half-life of a nuclide is the time taken for half of an initially large number of unstable nuclei to decay.

The larger the probability of decay per unit time, the shorter the half-life of the nuclide. When $t = t_{\frac{1}{2}}$ half the nuclei have decayed, so

$$N = \frac{N_0}{2}$$

But $N = N_0 e^{-\lambda t}$ so $e^{-\lambda t_{\frac{1}{2}}} = \frac{1}{2}$

Taking logarithms of both sides,

$$-\lambda t_{\frac{1}{2}} = \ln\tfrac{1}{2} = -\ln 2$$

gives

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

This is a very useful link.

The number of nuclei remaining after any particular time can be calculated using the decay equation. It can also be calculated using the half-life: if the time is a multiple of the half-life, e.g. n half-lives, then the number remaining is simply $1/2^n$ of the original number.

More generally, the fraction remaining after time t is given by $1/2^{t/t_{\frac{1}{2}}}$ (this can be derived from the decay equation).

Worked example

Americium-241 has a half-life of 433 y. (a) What is its decay constant? (b) What is the total activity of 1 g of this nuclide? (c) How much of this 1 g sample remains after 1000 y?

(a) $\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{\ln 2}{433 \times 3.16 \times 10^7\,\text{s}} = 5.1 \times 10^{-11}\,\text{s}^{-1}$

(b) In 1 g there are approximately (1 g/241 g) × 6.0×10^{23} atoms and the same number of nuclei, hence activity,

$A = \lambda N = 5.1 \times 10^{-11}\,\text{s}^{-1} \times \frac{1\,\text{g}}{241\,\text{g}} \times 6.0 \times 10^{23} = 1.3 \times 10^{11}$ Bq

(c) Fraction remaining = $e^{-\lambda t}$ = [illegible]

Summary of useful equations

Rate of decay	$\frac{dN}{dt} = -\lambda N$
Activity	$A = -\frac{dN}{dt} = \lambda N$
Decay equation	$N = N_0 e^{-\lambda t}$
Half-life	$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

PRACTICE

1 The activity of 20 g of element X is four times the activity of 10 g of element Y. Element Y has a half-life of 20 000 y.

a What is the half-life of X?

b How much of each will remain after i 40 000 y and ii 5000 y?

c How long will it be before equal amounts of X and Y remain?

2 Iodine-131 has a half-life of 8 days.

a What is the decay constant for iodine-131?

b What is the activity of 1 milligram of iodine-131?

c How long will it take for the activity of a 1 milligram sample of iodine-131 to fall to 10% of its original value?

3 Iodine-131 (discussed in question 2) is used as a tracer to check whether the thyroid gland is taking up iodine from the blood as it should. It emits both beta particles and gamma rays, and the latter are monitored from outside the body close to the thyroid gland. A dose of iodine of activity [illegible] Bq is injected into a patient's blood, and 20% of this is absorbed by the thyroid gland. The detector records 0.4% of the gamma-rays emitted.

a Why are the beta particles not monitored?

b Suggest reasons why the detector only records 0.4% of the gamma rays emitted from iodine-131 nuclei inside the thyroid.

c What activity would you expect the detector to record i soon after absorption, ii 16 days later, and iii three weeks after injection? State any assumptions.

4 Schools sometimes use 5 microcurie strontium-90 sources as beta sources. Strontium-90 has a half-life of 28 years. What is the total activity of a strontium-90 source 10 years after purchase?

5 The activity of a particular source falls from 5 × [illegible] Bq to 2 × [illegible] Bq in 20 minutes.

a What is the half-life and decay constant for this nuclide?

b How many atoms were there in the original source?

Radioactivity:

1. What is radioactivity?

Prirodzená rádioaktivita

Posunovacie pravidlá:

Premena α:

$${}^{A}_{Z}X - {}^{4}_{2}\alpha = {}^{A-4}_{Z-2}Y$$

Premena β^+

$${}^{A}_{Z}X - {}^{0}_{1}e = {}^{A}_{Z-1}Y + \nu$$

Premena β^-:

$${}^{A}_{Z}X - {}^{0}_{-1}e = {}^{A}_{Z+1}Y + \hat{\nu}$$

Premena γ:

$${}^{A}_{Z}X - \gamma = {}^{A}_{Z}X^*$$

${}^{0}_{-1}e = elektrón,\ {}^{0}_{1}e = pozitrón,\ \nu = neutrino,\ \hat{\nu} = antineutrino$

Zákon rádioaktívnej premeny:

N_t = počet nerozpadnutých jadier

$$N_t = N_0.e^{-\lambda t}, \quad \lambda = \frac{\ln 2}{T} = \frac{0{,}693}{T}$$

T = polčas rozpadu je čas, za ktorý sa rozpadne práve polovica jadier.

Zákon rádioaktívnej premeny:

N_t' = počet rozpadnutých jadier

$$N'_t = N_0.\lambda.t$$

$$N'_t \doteq \frac{m.N_A}{M_m}\cdot\frac{\ln 2}{T}\cdot t \doteq \frac{m}{Am_u}\cdot\frac{\ln 2}{T}\cdot t$$

2. Find the type of radioactive decay of the thorium:

$${}^{232}_{90}Th \rightarrow {}^{228}_{88}Ra \rightarrow {}^{228}_{89}Ac \rightarrow {}^{228}_{90}Th \rightarrow {}^{224}_{88}Ra \rightarrow {}^{220}_{86}Rn \rightarrow$$
$$\rightarrow {}^{216}_{84}Po \rightarrow {}^{212}_{82}Pb \rightarrow {}^{212}_{83}Bi \rightarrow {}^{212}_{84}Po \rightarrow {}^{208}_{82}Pb$$

3. Find the name of the isotope of an element which is created by four alfa and one beta decays from ${}_{92}{}^{238}U$.

4. Substitute the question marks in the equations:

$a.)\ {}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + ?$

$b.)\ {}^{55}_{25}Mn + ? \rightarrow {}^{55}_{26}Fe + {}^{1}_{0}n$

$c.)\ {}^{212}_{83}Bi \rightarrow {}^{208}_{81}Tl + ?$

$d.)\ {}^{212}_{83}Bi \rightarrow\ ? + {}^{0}_{-1}e$

$e.)\ {}^{30}_{15}P \rightarrow\ ? + {}^{0}_{1}e^+$

5. Find the halftime of the uranium if you its the decay constant is $\lambda = 4{,}33.10^{-4}year^{-1}$.
6. A nuclide has lifetime of 2400 seconds. How many per cent will be decayed in 300 seconds?

7. Number of decays in 1g of radium is $3{,}7.10^{10}s^{-1}$. Calculate the halftime of premeny

rádia $^{226}_{88}Ra$.

8. What is the time the half of the sample of radionuclide decays, if its decay constant is $\lambda = 1{,}42.10^{-11}s^{-1}$.
9. Pri určovaní veku pohrebného člna z hrobky Sesostrita III. zistili, že koncentrácia uhlíka v dreve, z ktorého bol čln urobený, je približne $N = 0{,}645.N_0$. N_0 je koncentrácia uhlíka v súčasných živých organizmov. Určite vek pohrebného člna ak polčas rozpadu uhlíka je 5730 rokov.
10. V septembri 1991 objavili turisti v tzalských Alpách v Rakúsku ľadom mumifikované ľudské telo. Uhlíkovou metódou sa zistilo, že pre nerozpadnuté jadrá v jeho tele N a jadrá v tele súčasných ľudí N platí $N = 0{,}5268N_0$. Pred koľkými rokmi tento človek zamrzol, keď polčas rozpadu uhlíka je T = 5730 rokov?