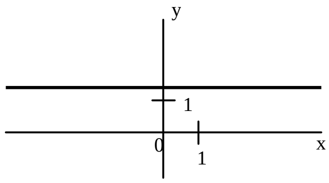
1. LINEAR FUNCTION $y = ax + b$, a = gradient, b- interception with *y-axis*

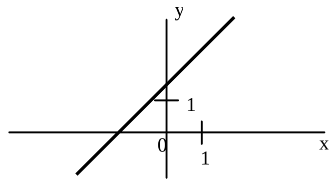
$\left(a = \frac{y_2 - y_1}{x_2 - x_1}\right)$

a = 0



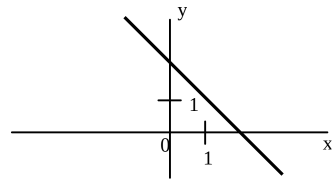
- Domain = R
- Range = {b}
- It's many to one,
- It's not one to one and therefore it's not increasing neither decreasing
- Bounded
- In each point $x \in R$ there is maximum and minimum

a > 0



- Domain = R
- Range = R
- It's one to one
- It's increasing
- Not bounded above, neither bounded below
- It has no maximum neither minimum

a < 0

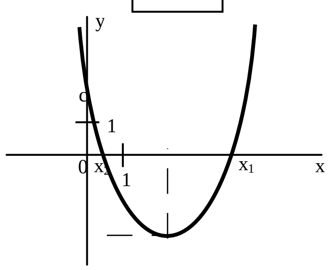


- Domain = R
- Range = R
- It's one to one
- It's decreasing
- Not bounded above, neither bounded below
- It has no maximum neither minimum

2. QUADRATIC FUNCTION $y = ax^2 + bx + c$, the graph of this function is **parable**

Useful formulas: Discriminant $D = b^2 - 4ac$ Roots: $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$ MAX/MIN $\left[-\frac{b}{2a}, c - \frac{b^2}{4a}\right]$

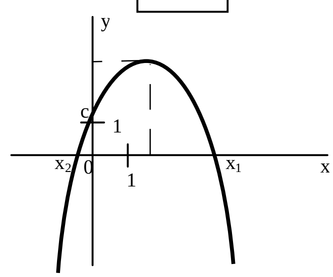
a > 0



- Domain = R – many to one
- Range = $\left\langle c - \frac{b^2}{4a}, \infty \right)$
- Decreasing $\left(-\infty, -\frac{b}{2a}\right\rangle$

  Increasing $\left\langle -\frac{b}{2a}, \infty \right)$
- Bounded below, not bounded above
- In point $x = -\frac{b}{2a}$ there is local minimum

a < 0



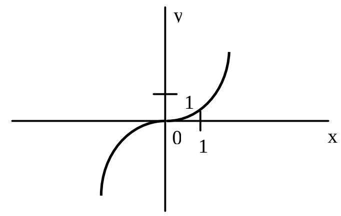
- Domain = R – many to one
- Range = $\left(-\infty, c - \frac{b^2}{4a}\right\rangle$
- Increasing $\left(-\infty, -\frac{b}{2a}\right\rangle$

  Decreasing $\left\langle -\frac{b}{2a}, \infty \right)$
- Bounded above, not bounded below
- In point $x = -\frac{b}{2a}$ there is local maximum

Worked out by Katarína Jakubíková

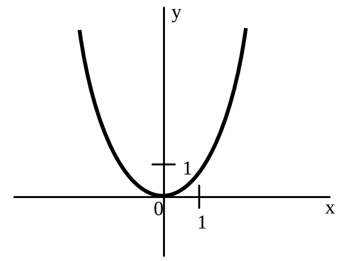
3. FUNCTION WITH NATURAL EXPONENT $y = x^n; n \in N$

n - uneven



- Domain = R
- Range = R
- Increasing
- One to one function
- Odd
- Not bounded below, neither above
- No minimum, No maximum

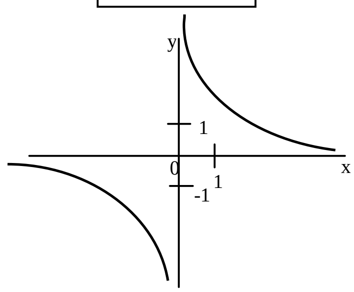
n - even



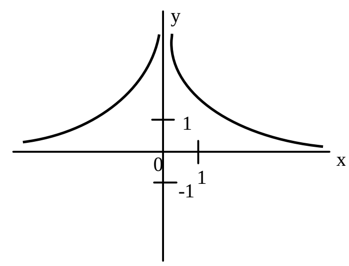
- Domain = R
- Range = R
- Decreasing $(-\infty, 0\rangle$
  Increasing $\langle 0, \infty)$
- Bounded below, not bounded above
- Even
- In point $x = 0$ there is local minimum, there is no maximum

4. FUNCTION WITH NEGATIVE EXPONENT $y = x^{-n}; n \in N$

n - odd



n - even



Domain = R- {0}

n - odd:

- Range = R- {0}
- Decreasing on the intervals $(-\infty, 0), (0, \infty)$
- Not bounded above neither below
- Odd

n - even:

- Range = R^+
- Decreasing on the interval $(-\infty, 0)$
  Increasing on the interval $(0, \infty)$
- Not bounded above, Bounded below
- Even

No maximum, no minimum

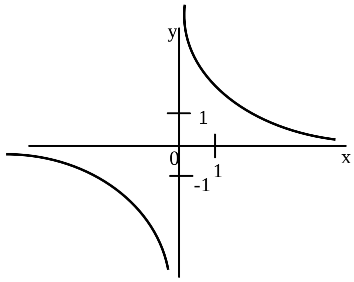
5. RATIONAL FRACTIONAL FUNCTION $y = \frac{P(x)}{Q(x)} = \frac{ax+b}{cx+d} = \frac{k}{x}; (k \neq 0)$

P(x) and Q(x) are polynomials and $Q(x) \neq 0$

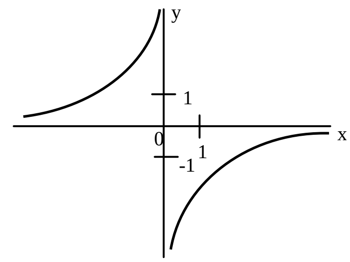
$a, b, c, d \in R$; $c, d \neq 0$ and $ad \neq bc$;(if $ad \neq bc$ then it is constant function)

- The graph of this function is **hyperbola**

k > 0



k < 0



Domain = R- {0}

Range = R- {0}

Decreasing on the intervals $(-\infty, 0), (0, \infty)$

Increasing on the intervals $(-\infty, 0), (0, \infty)$

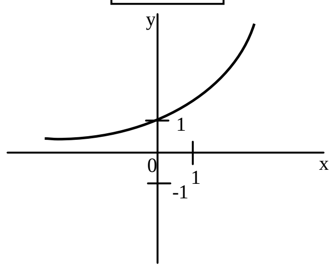
Not bounded above neither below

No maximum, no minimum

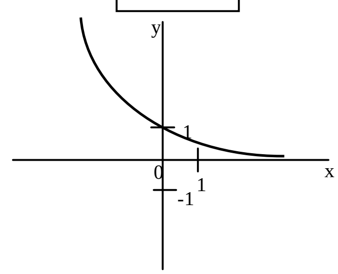
Odd

6. EXPONENTIAL FUNCTION $y = a^x$ (conditions: $a > 0; a \neq 1; x \in R$)

a > 0



0<a < 1



Domain = R

Range = $(0, \infty)$

Increasing and one to one

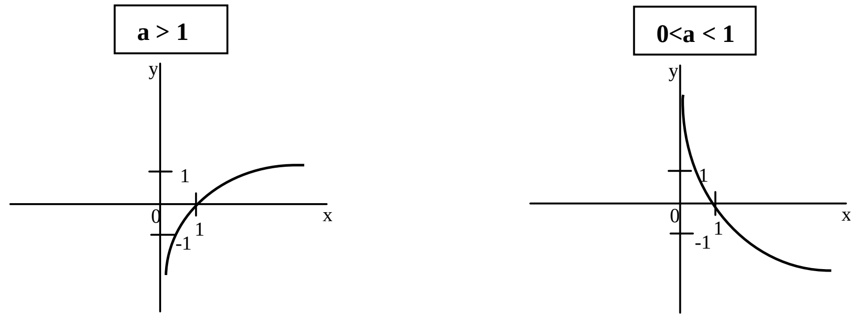
Decreasing and one to one

Bounded below, not bounded above

No minimum, no maximum

The image in point $x = 0$ is 1

7. LOGARITHMIC FUNCTION $y = \log_a x$



Domain = $(0,\infty)$

Range = R

Increasing and one to one — Decreasing and one to one

Not bounded below, not bounded above

No minimum, no maximum

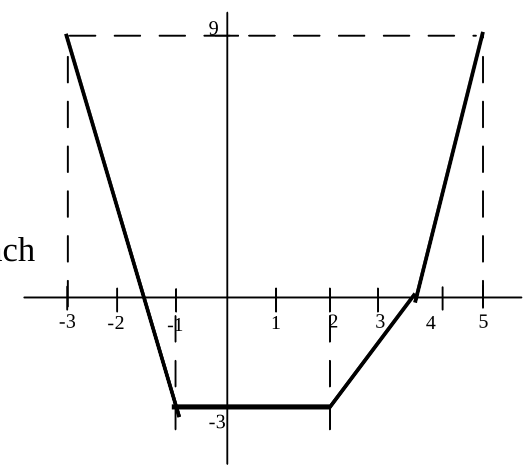
The image in point x = 1 is 0

An exponential function with the base e (Eller's number) is called **natural logarithmic function** y = ln$x$

8. MODULUS FUNCTION

We will use our knowledge about the linear function.

Example: Draw graph of the following function $y = |2-x| + |2x-7| + 3|1+x| - 15$; where $x \in \langle -3{,}5 \rangle$



1. find the critical points

   $2 - x = 0 \Rightarrow x = 2$

   $2x - 7 = 0 \Rightarrow x = \frac{7}{2}$

   $1 + x = 0 \Rightarrow x = -1$

2. Calculate the modulus expressions in each interval
3. Draw the graph

	$\langle -3,-1 \rangle$	$\langle -1{,}2 \rangle$	$\langle 2;3{,}5 \rangle$	$\langle 3{,}5;5 \rangle$
|2-x| |2x-7| |1+x| 3|1+x|	2-x 7-2x -1-x -3-3x	2-x 7-2x 1+x 3+3x	x-2 7-2x 1+x 3+3x	x-2 2x-7 1+x 3+3x
$\lvert 2-x\rvert + \lvert 2x-7\rvert + 3\lvert 1+x\rvert - 15$	-6x-9	-3	2x-7	6x-21
	$y_1 = -6x-9$ $x \in \langle -3,-1 \rangle$	$y_2 = -3$ $x \in \langle -1{,}2 \rangle$	$y_3 = 2x-7$ $x \in \langle 2;3{,}5 \rangle$	$y_1 = 6x-21$ $x \in \langle 3{,}5;5 \rangle$