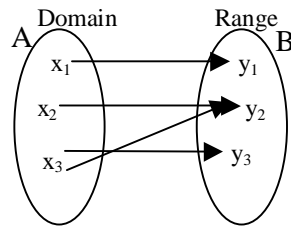
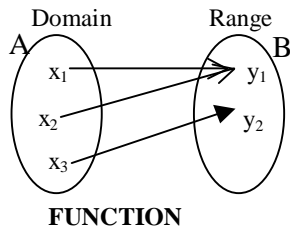


FUNCTION AND PROPERTIES OF FUNCTION – SUMMARY

WHAT IS FUNCTION?

Function is the relationship of two sets (A, B), such that for each element in A there exists only one image in the set B. ($A, B \in \mathbb{R}$)



NOT A FUNCTION – there are 2 images in set B for one point in the set A (ONE TO MANY)

DEFINITION OF THE FUNCTION

Every set of ordered pairs $[x, y] \in \mathbb{R} \times \mathbb{R}$ for which is valid: for each element $x \in \mathbb{R}$ there exists **just one** $y \in \mathbb{R}$, so that $[x, y] \in f(x)$.

DOMAIN OF A FUNCTION – we denote set A as $D(f)$ and we call it the **DOMAIN** of the function.

Notation: $D(f) = \{x \in \mathbb{R}; \exists y \in \mathbb{R}; y \in f(x)\}$

RANGE OF A FUNCTION – we denote set B as $R(f)$ and call it the **RANGE** of a function $f(x)$. For each element in set B there exists at least one element in set A.

(In Slovakia we denote $R(f)$ as $H(f)$ – obor hodnôt)

Notation: $R(f) = \{y \in \mathbb{R}; \exists x \in \mathbb{R}; y \in f(x)\}$

PROPERTIES OF THE FUNCTION

INCREASING FUNCTION (rastúca)

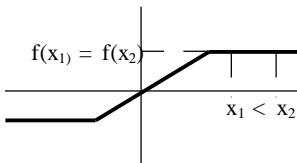
$f(x)$ is a function and x_1 and $x_2 \in D(f)$; $f(x)$ is **increasing** if $x_1 < x_2$ implies that $f(x_1) < f(x_2)$

DECREASING FUNCTION (klesajúca)

$f(x)$ is a function and x_1 and $x_2 \in D(f)$; $f(x)$ is **decreasing** if $x_1 < x_2$ implies that $f(x_1) > f(x_2)$

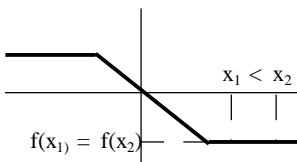
NON-DECREASING FUNCTION (neklesajúca)

$f(x)$ is a function and x_1 and $x_2 \in D(f)$; $f(x)$ is **non-decreasing** if $x_1 < x_2$ implies that $f(x_1) \leq f(x_2)$



NON-INCREASING FUNCTION (nerastúca)

$f(x)$ is a function and x_1 and $x_2 \in D(f)$; $f(x)$ is **non-increasing** if $x_1 < x_2$ implies that $f(x_1) \geq f(x_2)$



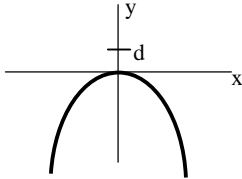
MANY-TO-ONE FUNCTION = **non-decreasing or non-increasing function or each function for which is valid that there exist at least two x with one f(x).**

ONE-TO-ONE FUNCTION (prostá)

$f(x)$ is a **one-to-one** function if for all x_1 and $x_2 \in D(f)$; such that $x_1 \neq x_2$ $f(x)$ implies that that $f(x_1) \neq f(x_2)$. (Exponential function is an example of one-to-one function)
(LINE TEST – function is one-to-one if and only if each horizontal line intersects the graph of the function just in one point)

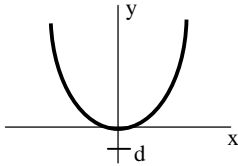
BOUNDED ABOVE FUNCTION (ohraničená z hora)

$f(x)$ is a function and $A \in D(f)$; $f(x)$ is **bounded above** if $\exists d \in \mathbb{R}$ such that $f(x) \leq d$ for all $x \in D(f)$



BOUNDED BELOW FUNCTION (ohraničená z dola)

$f(x)$ is a function and $A \in D(f)$; $f(x)$ is **bounded below** if $\exists d \in \mathbb{R}$ such that $f(x) \geq d$ for all $x \in D(f)$

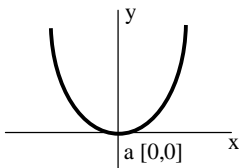


BOUNDED

Function is bounded if it is bounded below and bounded above as well.

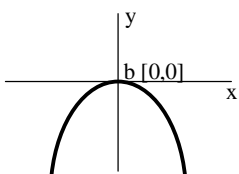
MINIMUM OF THE FUNCTION

Let $A \subseteq D(f)$ and $a \in A$; the function has **minimum** at point $a \in A$ if for all $x \in A$ $f(x) \geq f(a)$
(we consider $f(x) > 0$)



MAXIMUM OF THE FUNCTION

Let $A \subseteq D(f)$ and $b \in A$; the function has **maximum** at point $b \in A$ if for all $x \in A$ $f(x) \leq f(b)$
(we consider $f(x) < 0$)



EVEN FUNCTION (párna)

Function f is **even** just then if it is valid that for each $x \in D(f)$ exists also $-x \in D(f)$ such that $f(-x) = f(x)$.

Function is symmetric around the y-axis. (Example of even function is $y = x^2$)

ODD FUNCTION (nepárna)

Function f is **odd** just then if it is valid for each $x \in D(f)$ exists also $-x \in D(f)$ such that $f(-x) = -f(x)$.

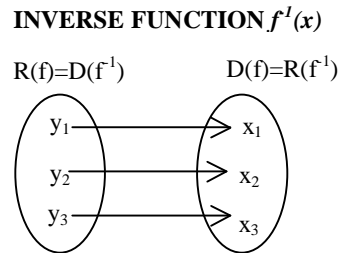
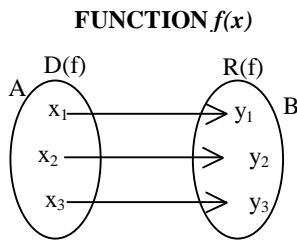
Function is symmetric around the origin. (Example of odd function is $y = x$)

INVERSE FUNCTION

Let $f(x)$ to be an **one-to-one** function with $D(f)$ and $R(f)$; then we can find the **inverse function** to this function if for each $y \in R(f)$ we can assign $x \in D(f)$ for which $f(x) = y$

Notation of inverse function $f^{-1}(x)$, $g(x)$

Function and inverse function are symmetric around the line $y = x$



If function $f(x)$ is one-to-one function it has inverse function, which is one-to-one as well.
If function $f(x)$ is not one-to-one function it doesn't have inverse function.

EXAMPLE

Imagine a free fall and you want to know the distance in time 0; 1; 1,5...4 seconds. The formula for free fall is $s = \frac{1}{2} g \cdot t^2$ ($g = 10 \text{ ms}^{-2}$). We can see it's quadratic function, so the graph is parable.

t [s]	0	1	1,5	2	2,5	3	3,5	4
s [m]	0	5	11,25	20	31,25	45	61,25	80

Time t is the variable that implies $D(h) = \langle 0,4 \rangle$

Distance s creates the Range of this function $R(h) = \langle 0,80 \rangle$

$$h: s = \frac{1}{2} g \cdot t^2$$

The inverse function to function h is:

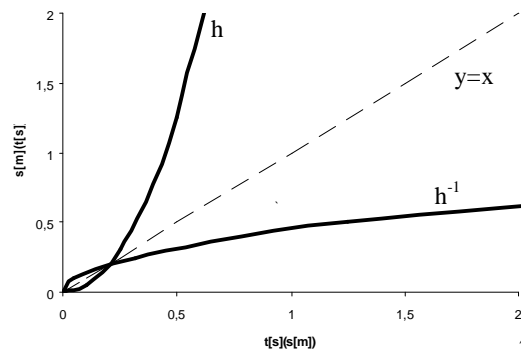
s [m]	0	1,25	5	11,25	20	31,25	45	61,25	80
t [s]	0	0,5	1	1,5	2	2,5	3	3,5	4

$$h^{-1}: t = \frac{1}{2} g \cdot s^2 \Rightarrow 2t = g \cdot s^2 \Rightarrow \frac{2t}{g} = s^2$$

$$s = \pm \sqrt{\frac{2t}{g}} \quad (\text{we can't have negative distance, so we consider only positive})$$

$$D(h^{-1}) = \langle 0,80 \rangle$$

$$R(f) = \langle 0,4 \rangle$$



Worked out by Katarína Jakubíková