

**Circle = circumference** is a set of points X in the plane which have the same distance **r** from the fixed point **S**.

$$|SX| = r = X - S$$

If  $X[x, y]$  and  $S[m, n]$ , then the analytical expression of a circumference with the centre S and radius r is:

$$(x - m)^2 + (y - n)^2 = r^2$$

This equation is called the *centre form of the circumference equation*. If we raise the brackets to the powers, we get the *general form of the circle equation*:  $x^2 + y^2 - 2mx - 2ny + m^2 + n^2 - r^2 = 0$ , what is in fact:  $x^2 + y^2 + ax + by + c = 0$ .

Basic terminology:

A **chord** is an abscissa that joins two points on the circumference of a circle.

A **tangent** to a circle is a straight line that touches the circle at one point only.

A **diameter** is a chord that passes through the centre.

**Circumference** is the distance measured around the curved edge of a circle.

A **radius** is any abscissa from the centre to the circumference.

The curved part of the circle between points A and B is known as an **arc**. There are two arcs AB: The larger is the **major arc**, the smaller is the **minor arc**.

The chord AB divides the circle into two **segments**, the major segment and the minor segment.

The **perpendicular bisector** of any chord passes through the centre of its circle (i.e. it is a line that is perpendicular to the chord and passes through its centre).

Any straight line that cuts across a circle at two points is called a **secant**. (A tangent is a special case of a secant, as the two cuts become a single point at contact. These two cuts are said to become coincident.)

## TANGENTS TO A CIRCLE

The angle between a radius, drawn to the point of contact of a tangent, and the tangent itself is a right angle.

From any point P, outside the circle, two tangents can be drawn; the distance from P to the point of contact being the same for each tangent.

If point X lies:

- a) On the circumference, then  $(x - m)^2 + (y - n)^2 = r^2$
- b) Inside the circle, then  $(x - m)^2 + (y - n)^2 < r^2$

c) Outside the circle, then  $(x - m)^2 + (y - n)^2 > r^2$

### Mutual position of a circle and other linear shapes

Ex: Circle k has a centre S[2, 3] and  $r = 5$ . You're given a point A[-3, -4] and B[1, 6]. Find the intersection of AB and the circle k.

We need to write the centre form of circle equation:  $(x - 2)^2 + (y - 3)^2 = 25$ , and the vector equations of a line AB:  $x = -3 + 4t$ ,  $y = -4 + 10t$ .

We substitute for x, and y into the quadratic equation and we get a value of parameter t.

If AB is an abscissa then  $t \in \langle 0, 1 \rangle$ .

### Tangent to a circle

- Contains one point of a circle only
- The centre of the circle has a distance of r from it
- Is perpendicular to that radius of a circle, which contains the point of contact

The equation  $(x_0 - m)(x - m) + (y_0 - n)(y - n) = r^2$  is the analytical expression of a tangent to a circle  $(x - m)^2 + (y - n)^2 = r^2$  at a point T[ $x_0, y_0$ ].