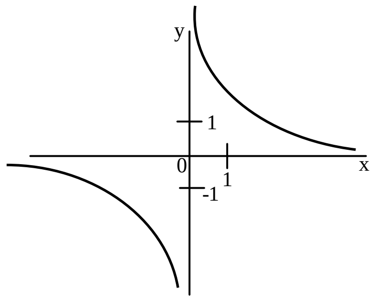
Linear fractional function

$$y = \frac{P(x)}{Q(x)} = \frac{ax+b}{cx+d} = \frac{k}{x};(k \neq 0)$$

P(x) and Q(x) are polynomials and $Q(x) \neq 0$, $a,b,c,d \in R; c,d \neq 0$ and $ad \neq bc$;(if $ad \neq bc$ then it is constant function)

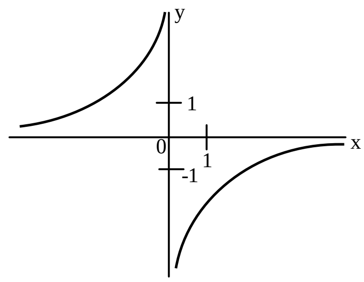
- The graph of this function is **hyperbola**

k > 0



Decreasing on the intervals $(-\infty,0),(0,\infty)$

k < 0



Increasing on the intervals $(-\infty,0),(0,\infty)$

Domain = R- {0}
Range = R- {0}

Not bounded above neither below
No maximum, no minimum
Odd

Example

Draw the graph of the function $y = \frac{3-x}{x-1}$ and write the properties.

Divide (*3-x)* by (*x* – 1)

Look just at the leading *x* of the divisor and the leading -x of the dividend.

$(-x+3) \div (x-1) = -1$

Now I'll take that *x*, and multiply it through the divisor, *x* - 1.

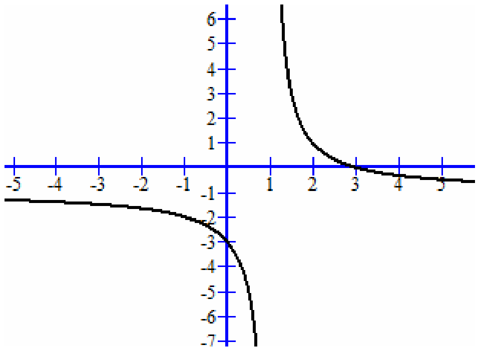
$$\begin{array}{l} \underline{(-x+3)} \div (x-1) = -1 \\ -x+1 \end{array}$$

To <u>subtract</u> the polynomials, **I *change all the signs* in the second line**

$$\begin{array}{l} \underline{(-x+3)} \div (x-1) = -1 \\ \underline{-(-x+1)} \\ 0+2 \end{array}$$

So I can write $y = \frac{3-x}{x-1} = -1 + \frac{2}{x-1}$

D(f) = R - {1}; H(f) = R - {-1},
decreasing, not even, not odd, one-to-one, not bounded, no Min, no Max



Exercise

Draw the graphs of the following functions and write their properties

a) $y = \frac{2x+3}{x-1}$,

b) $y = \frac{-x+2}{x-1}$,

c) $y = \frac{2x-4}{-x+2}$

d) $y = \frac{x+3}{x-1}$,

e) $y = \frac{5-2x}{3x-1}$

f) $y = \frac{3x+3}{2x-4}$