

POWER FUNCTIONS in  $x$  are of the form  $x^n$ . If  $n > 0$ , the graph of  $y = x^n$  is said to be of the parabolic type (the curve is a parabola for  $n = 2$ ). If  $n < 0$ , the graph of  $y = x^n$  is said to be of the hyperbolic type (the curve is a hyperbola for  $n = -1$ ).

**1. Function with positive natural exponent  $y = x^n; n \in \mathbb{N}$**

**A) even exponent**

Domain =  $\mathbb{R}$

Range =  $(0, \infty)$

Many – to – one function

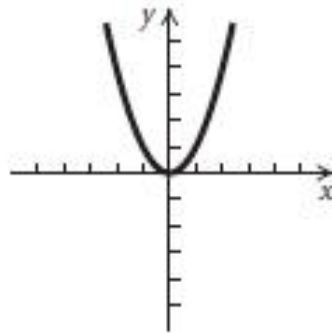
Decreasing  $(-\infty, 0)$

Increasing  $(0, \infty)$

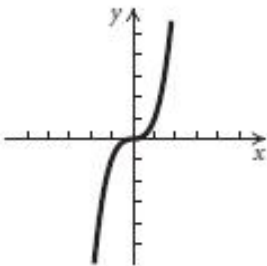
Bounded from below

Even

There is a local minimum at the point  $x = 0$



**B) odd exponent**



Domain =  $\mathbb{R}$

Range =  $\mathbb{R}$

One to one function

Increasing

Not bounded

Odd

No minimum, No maximum

**2. Function with negative integer exponent  $y = x^{-n}; n \in \mathbb{Z}$**

**A) even integer**

Domain =  $\mathbb{R} - \{0\}$

Range =  $(0, \infty)$

Many – to- one function

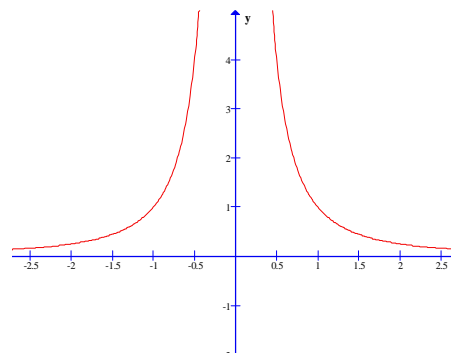
Increasing  $(-\infty, 0)$

Decreasing  $(0, \infty)$

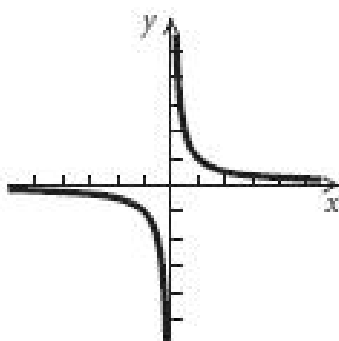
Bounded from below (lower limit)

Even

No minimum, No maximum



**B) odd integer**



Domain =  $\mathbb{R} - \{0\}$

Range =  $\mathbb{R} - \{0\}$

One – to- one function

Decreasing on the intervals  $(-\infty, 0) \cup (0, \infty)$

Not Bounded

Odd

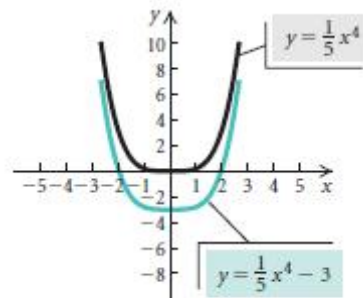
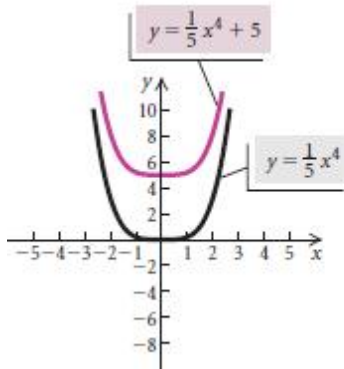
No minimum, No maximum

### Vertical Translation

For  $b > 0$ ,

the graph of  $y = f(x) + b$  is the graph of  $y = f(x)$  shifted *up*  $b$  units;

the graph of  $y = f(x) - b$  is the graph of  $y = f(x)$  shifted *down*  $b$  units.

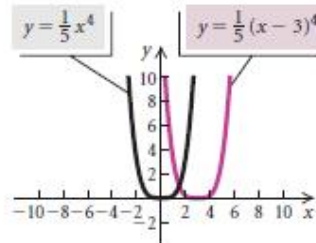
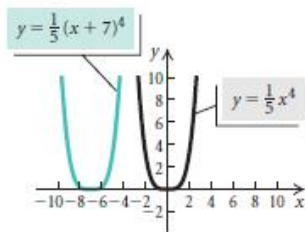


### Horizontal Translation

For  $d > 0$ :

the graph of  $y = f(x - d)$  is the graph of  $y = f(x)$  shifted *right*  $d$  units;

the graph of  $y = f(x + d)$  is the graph of  $y = f(x)$  shifted *left*  $d$  units.

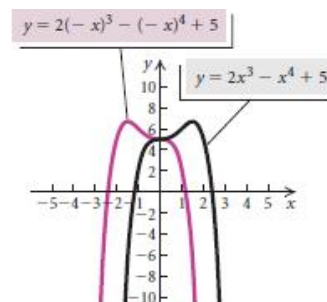
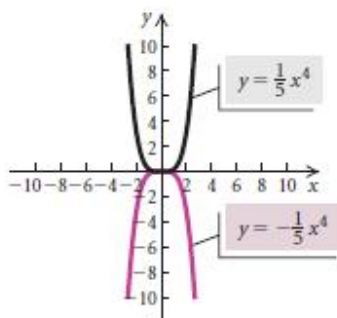


### Reflections

The graph of  $y = -f(x)$  is the **reflection** of the graph of  $y = f(x)$  across the  $x$ -axis.

The graph of  $y = f(-x)$  is the **reflection** of the graph of  $y = f(x)$  across the  $y$ -axis.

If a point  $(x, y)$  is on the graph of  $y = f(x)$ , then  $(x, -y)$  is on the graph of  $y = -f(x)$ , and  $(-x, y)$  is on the graph of  $y = f(-x)$ .

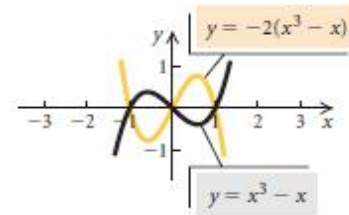
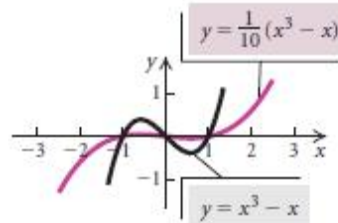
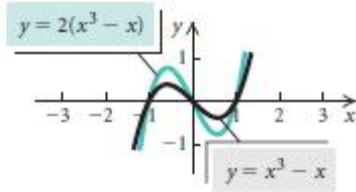


### Vertical Stretching and Shrinking

The graph of  $y = af(x)$  can be obtained from the graph of  $y = f(x)$  by

- stretching vertically for  $|a| > 1$ , or
- shrinking vertically for  $0 < |a| < 1$ .

For  $a < 0$ , the graph is also reflected across the  $x$ -axis.  
(The  $y$ -coordinates of the graph of  $y = af(x)$  can be obtained by multiplying the  $y$ -coordinates of  $y = f(x)$  by  $a$ .)

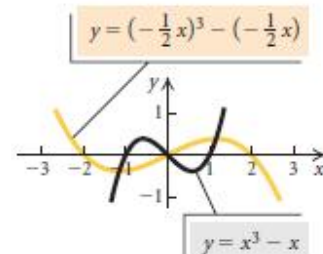
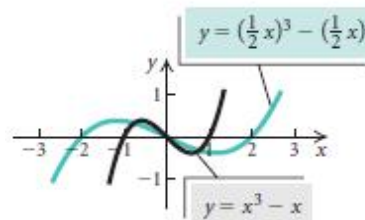
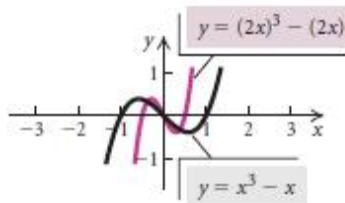


### Horizontal Stretching and Shrinking

The graph of  $y = f(cx)$  can be obtained from the graph of  $y = f(x)$  by

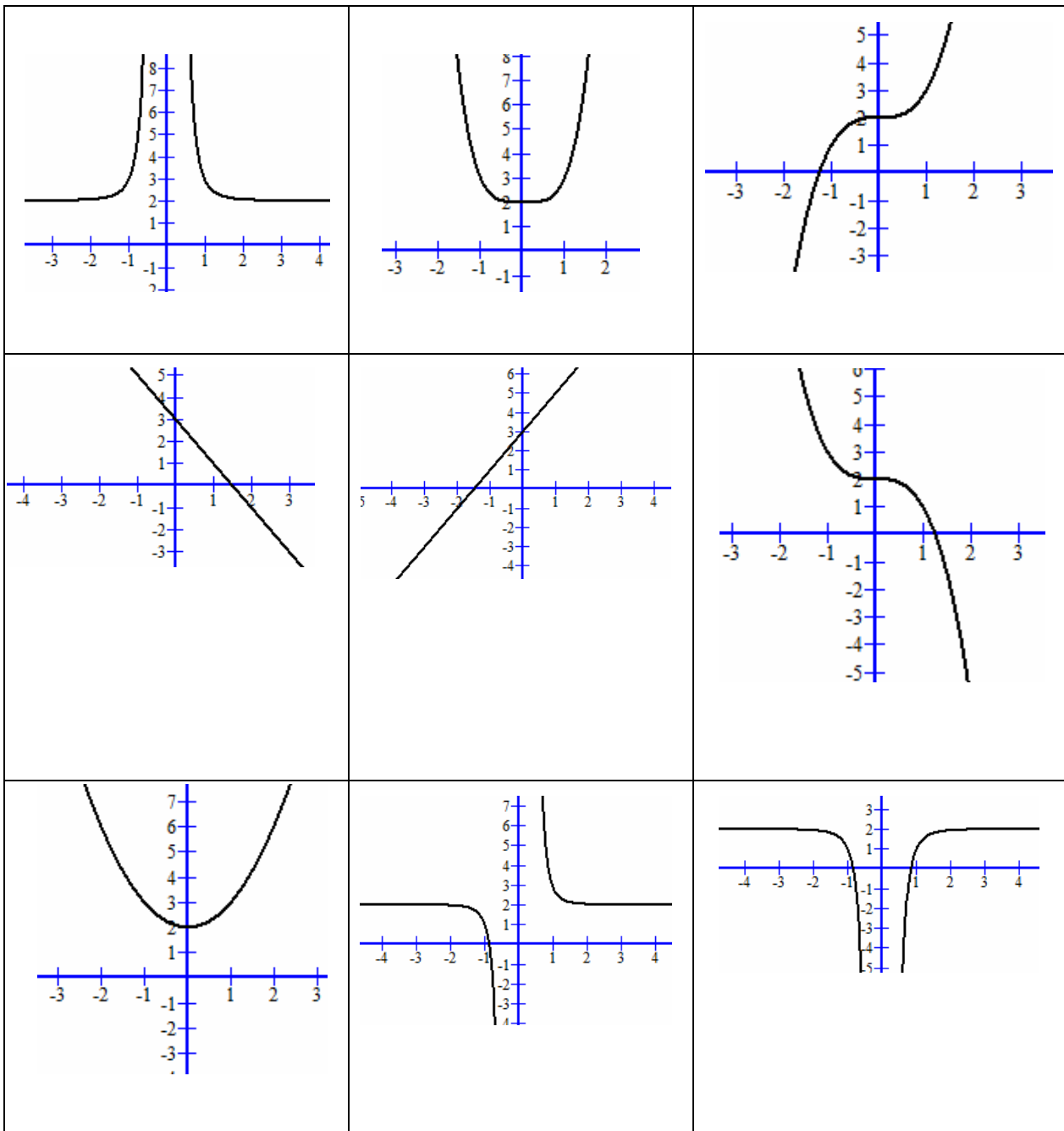
- shrinking horizontally for  $|c| > 1$ , or
- stretching horizontally for  $0 < |c| < 1$ .

For  $c < 0$ , the graph is also reflected across the  $y$ -axis.  
(The  $x$ -coordinates of the graph of  $y = f(cx)$  can be obtained by dividing the  $x$ -coordinates of the graph of  $y = f(x)$  by  $c$ .)



Exercise:

Match the graphs with the corresponding equations



1. $y = 2x + 3$	2. $y = x^4 + 2$	3. $y = x^{-3} + 2$	4. $y = -x^4 + 2$	5. $y = x^3 + 2$
6. $y = x^{-4} + 2$	7. $y = -2x + 3$	8. $y = x^2 + 2$	9. $y = -x^3 + 2$	