

**LINEAR FUNCTIONS**

Definition: **Linear function** is each function given by formula  $f: y = mx + c$ ;  $m, c \in \mathbb{R}$ ;  $D(f) = \mathbb{R}$

If  $x = 0$ , then  $y = c$ , i.e.  $c$  is the functional value of the function at point 0 ( $f(0) = c$ )

Linear function given by formula  $y = c$  is a **constant function**. A graph of this function is a straight-line parallel to x-axis, cutting the y-axis at point  $[0, c]$ .

It's valid:  $m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$ , where  $m$  is the gradient = slope of the line which shows us, whether

1. the function/line is increasing  $\Leftrightarrow m > 0$
2. the function/line is decreasing  $\Leftrightarrow m < 0$
3. the function/line is constant  $\Leftrightarrow m = 0$

**Ex:** Find all linear functions that contains the ordered pairs  $[0,2]$  and  $[-3,5]$

Formula of linear function is  $y = mx + c$ , where  $m, c$  are unknown constants now. We have to substitute into the formula for  $x$  and  $y$ . Then we have to solve the simultaneous system of equations.

$$y = mx + c$$

$$2 = m \cdot 0 + c \Rightarrow c = 2$$

$$\underline{5 = m \cdot (-3) + c}$$

$$5 = -3m + 2$$

$$m = -1$$

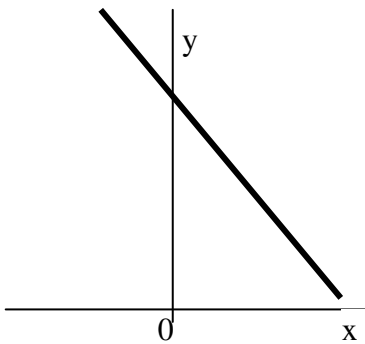
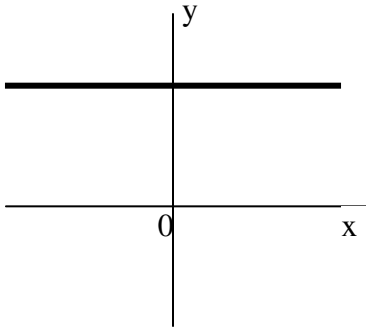
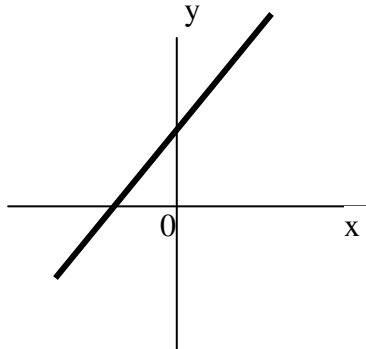
$$f: y = -x + 2$$

Other way of solving this is to calculate gradient  $m$  firstly according to the formula:

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{5 - 2}{-3 - 0} = -1$$

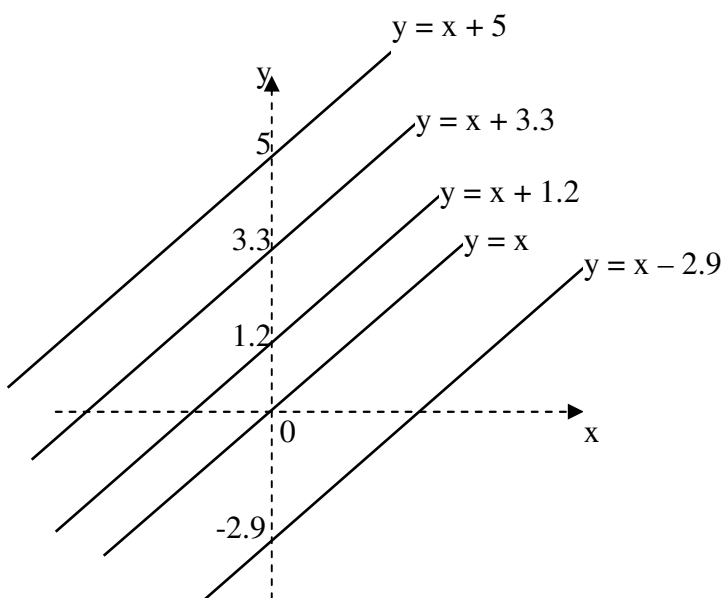
So the linear function is  $y = -x + 2$  because for  $x = 0$  is  $y = -0 + 2 = 2$ .

**PROPERTIES OF LINEAR FUNCTION**  $f: y = mx + c$

If $m < 0$	If $m = 0$	If $m > 0$
Graph: 	Graph: 	Graph: 
Domain: $D(f) = \mathbb{R}$	Domain: $D(f) = \mathbb{R}$	Domain: $D(f) = \mathbb{R}$
Range of values: $H(f) = \mathbb{R}$	Range of values: $H(f) = \{b\}$	Range of values: $H(f) = \mathbb{R}$
Decreasing function – for each $x$ : If $x_1 < x_2$ so $f(x_1) > f(x_2)$	Not increasing, not decreasing – for each $x$ : If $x_1 < x_2$ so $f(x_1) = f(x_2) = c$	Increasing function – for each $x$ : If $x_1 < x_2$ so $f(x_1) < f(x_2)$
One-to-one function – for each $x$ : If $x_1 \neq x_2$ so $f(x_1) \neq f(x_2)$	Many-to-one function – for each $x$ : If $x_1 \neq x_2$ so $f(x_1) = f(x_2) = c$	One-to-one function – for each $x$ : If $x_1 \neq x_2$ so $f(x_1) \neq f(x_2)$
Unbounded function	Bounded function	Unbounded function
Function has neither maximum nor minimum	Function has a maximum and minimum at each point $x \in D(f)$	Function has neither maximum nor minimum

**Ex:** We have a set  $S$  of all linear functions according to  $y = x + c$  if  $c \in \mathbb{R}$ . Draw graphs of few of these functions.

Graphs of functions from set  $S$  are straight-lines parallel to straight-line defined by function  $y = x$  and  $c$  is a parameter that defines distance from the origin of coordinate system.



## LINEAR FUNCTION

Mgr. Ľubomíra Tomková

**Ex:** We have a set S of all linear functions according to  $y = mx + 3$  if  $m \in \mathbb{R}$ . Draw graphs of few of these functions.

Graphs of functions from set S are straight-lines that pass through the point  $[0,3]$ . Parameter  $m$  defines the slope of straight-lines.

