

**MATRICES**

A matrix stores mathematical information in a concise way. The information is written down in a rectangular array of rows and columns of terms, called **elements** or **entries**, each of which has its own precise position in the array.

Notation: We normally represent matrices by capital letters. For example A, B, M, etc.

**The order of a matrix** is its shape.

- ⇒ When stating the order of a matrix, we must always give first the number of rows, followed by the number of columns.
- ⇒  $\begin{pmatrix} 4 \\ 8 \\ 7 \end{pmatrix}$  is a **column** matrix and has order 3 x 1, since its elements are arranged in three rows and only one column.
- ⇒ the matrix (4 8 7) has order 1 x 3 and is a **row** matrix.
- ⇒ when the number of rows and the number of the columns are equal, the matrix is called a square matrix, i.e. it has order n x n.

**Addition and Subtraction of Matrices**

We can add or subtract two matrices only when they are of the *same order*. We add or subtract an element from matrix A with that element from matrix B, which is on the same position.

**Multiplication of matrices**

- ⇒ To multiply a matrix by any real number **k** we multiply every element of the matrix by **k**.
- ⇒ To multiply a matrix by another one we have to take into account the following rule: The number of columns in the first matrix must be the same as the number of rows in the second matrix.

To multiply **A** by **B**, we start by taking the first row of matrix **A** (2 3 1) and the first

column of matrix **B**  $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ .

We then multiply the first element of the row by the 1<sup>st</sup> element of the column, the 2<sup>nd</sup> element of the row by the 2<sup>nd</sup> element of the column, and the 3<sup>rd</sup> element of the row by

the last element of the column. We then add up these three products. This gives the element in the top left-hand corner of the matrix  $\mathbf{AB}$ , which is  $2.1 + 3.1 + 1.0 = 5$

Next we take the first row of matrix  $\mathbf{A}$  and the second column of matrix  $\mathbf{B}$ , which gives us the 2<sup>nd</sup> element of the first row of matrix  $\mathbf{AB}$ .

Generally, the product  $\mathbf{AB}$  produces a matrix which has the same number of rows as  $\mathbf{A}$ , and the same numbers of columns as  $\mathbf{B}$ . Hence, if  $\mathbf{A}$  has order  $a \times t$  and  $\mathbf{B}$  has order  $t \times b$ , then  $\mathbf{AB}$  has order  $a \times b$ .

- ⇒ Multiplication of two matrices is **not commutative**. That is,  $\mathbf{AB} \neq \mathbf{BA}$ . Therefore we must ensure that we write matrices in the correct sequence.
- ⇒ Multiplication of any three matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  is associative, i.e.  $\mathbf{A(BC)} = (\mathbf{AB})\mathbf{C}$