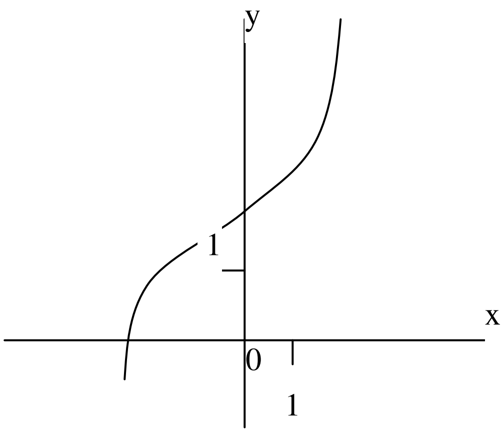
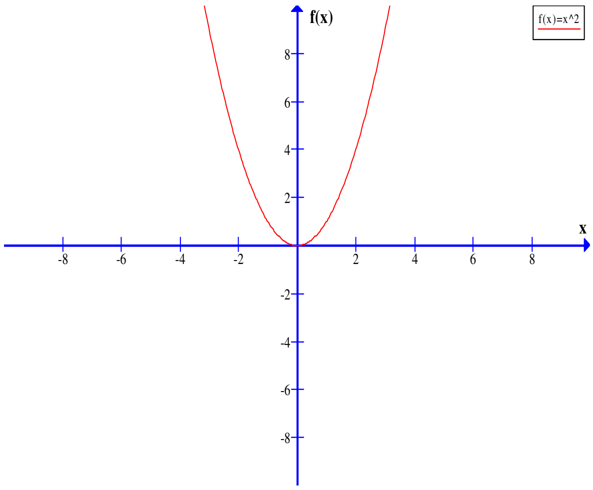
POWER FUNCTIONS

Power function is a function of the form: **f(x): $y = x^n$,** $n \in N$, where *x* is the *base* raised to the *power* of *n*.

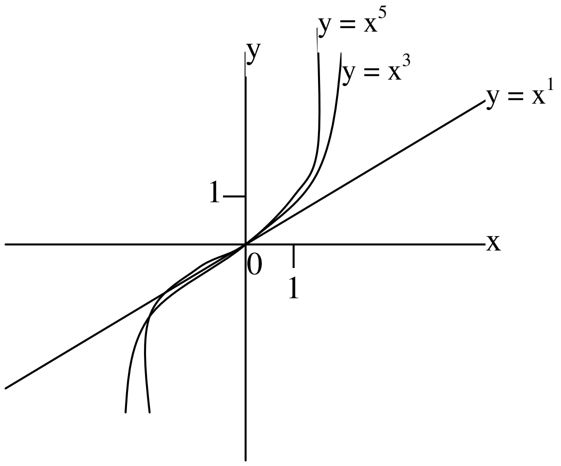
We distinguish positive and negative powers, moreover even and odd powers.

POWER FUNCTIONS WITH *POSITIVE* POWER

Power **n** is an **odd** number (n = 1, 3, 5 …)	Power **n** is an **even** number (n = 2, 4, 6 …)
The graph is a *cubic curve*: 	The graph is a *parabola*: 
Domain D(f) = R, D(f) = $(-\infty, \infty)$	Domain D(f) = R, D(f) = $(-\infty, \infty)$
Range of values H(f) = R, D(f) = $(-\infty, \infty)$	Range of values H(f) = $\langle 0, \infty)$
Increasing function on D(f) = $(-\infty, \infty)$	Increasing on $\langle 0, \infty)$, decreasing on $(-\infty, 0\rangle$
Odd function - for each x: f(-x) = - f(x)	Even function – for each x: f(x) = f(-x)
Not bounded function on D(f) = $(-\infty, \infty)$	Function bounded from below, not from above
no minimum, no maximum	Local minimum at point 0, no maximum

Ex. 1 Compare graphs of functions $y = x^1$, $y = x^3$, $y = x^5$.

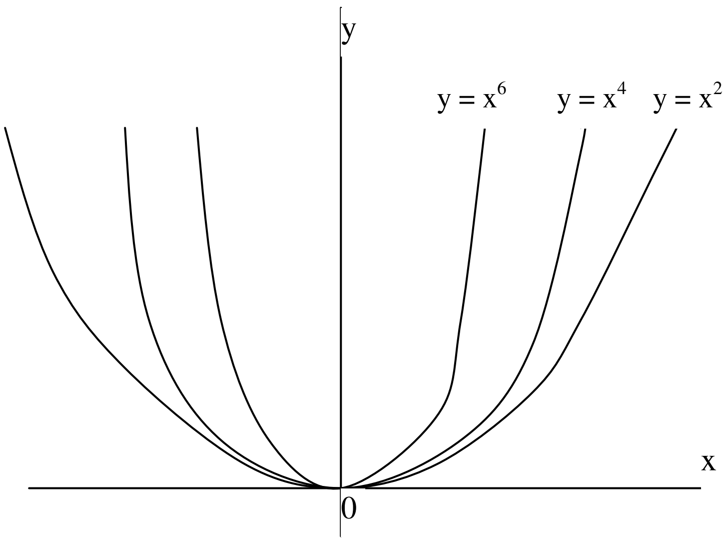
x	**- 1.5**	**-1**	**- 0.5**	**0**	**0.5**	**1**
$y = x^1$	- 1.5	-1	- 0.5	0	0.5	1
$y = x^3$	- 3.375	-1	- 0.125	0	0.125	1
$y = x^5$	- 7.594	-1	- 0.031	0	0.031	1



The power defines the slope of the graph. The higher power, the graph is nearer to y-axis.

Ex. 2 Compare graphs of functions $y = x^2$, $y = x^4$, $y = x^6$.

x	**- 1.5**	**-1**	**- 0.5**	**0**	**0.5**	**1**
$y = x^2$	2.25	1	0.25	0	0.25	1
$y = x^4$	5.063	1	0.063	0	0.063	1
$y = x^6$	11.391	1	0.016	0	0.016	1



REVISION :

Basic rules for calculating with powers:

$x^n \times x^m = x^{n+m}$ $\quad \dfrac{x^n}{x^m} = x^{n-m}$ $\quad (x^m)^n = x^{m \times n}$

$(x_1 \times x_2)^n = x_1^n \times x_2^n$ $\quad x^{-n} = \dfrac{1}{x^n} \quad x \neq 0$ $\quad x^{\frac{n}{m}} = \sqrt[m]{x^n} \quad x \geq 0$

POWER FUNCTIONS WITH *NEGATIVE* POWER

Formula: $y = x^{-n} = \frac{1}{x^n}$, $n \in N$

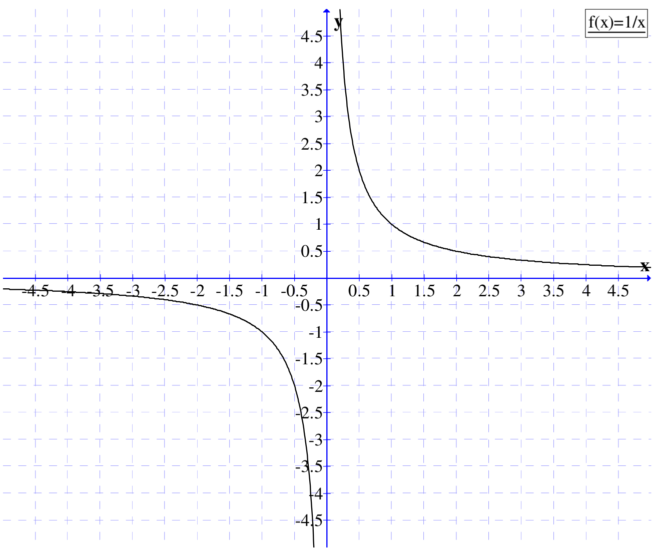
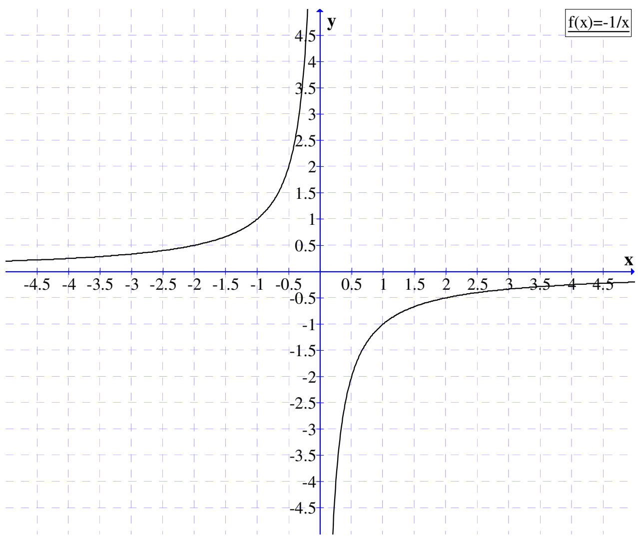
When the power is negative we apply the formula for calculation with a negative index, i.e. we will obtain a fraction, where in the denominator there will be the unknown x raised to positive power. Since the unknown x occurs now in the denominator, we need to consider the domain of the function – we know there cannot be zero in the denominator, i.e. the whole expression occurring in the denominator has the have its value different from 0.

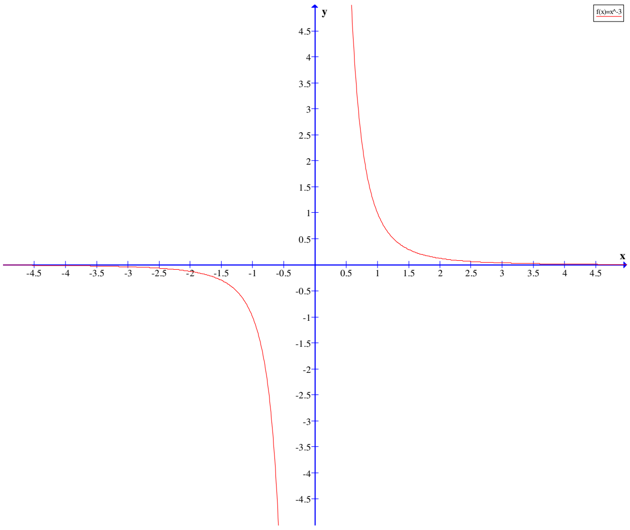
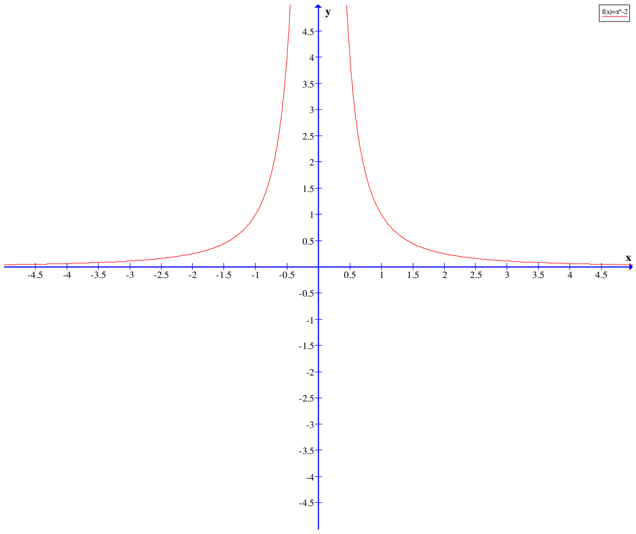
When the power is **– 1** the functions is called the indirect ratio/proportionality, i.e. $x^{-1} = \frac{1}{x}$.

INDIRECT PROPORTIONALITY

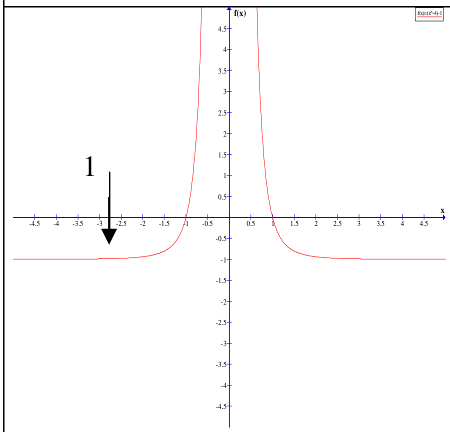
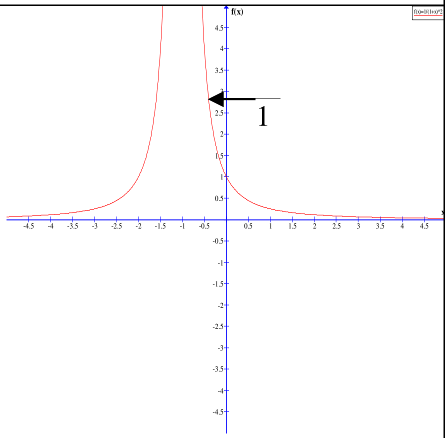
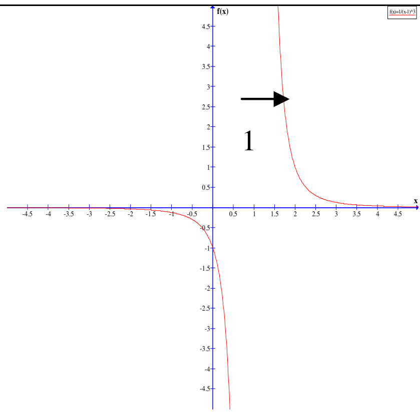
Definition: Let *k* be a real number, $k \neq 0$. **Indirect ratio** is the function given by formula $y = \frac{k}{x}$.

D(f) = R – {0}. A graph of this function is a rectangular *hyperbola*.

<table>
<tr><th>k > 0</th><th>k < 0</th></tr>
<tr><td>Graph:

</td><td>Graph:

</td></tr>
<tr><td>Domain D(f)= R–{0}, D(f)= $(-\infty,0)\cup(0,\infty)$</td><td>Domain D(f)= R–{0}, D(f)= $(-\infty,0)\cup(0,\infty)$</td></tr>
<tr><td>Range of values H(f)= R–{0}, D(f)= $(-\infty,0)\cup(0,\infty)$</td><td>Range of values H(f)= R–{0}, D(f)= $(-\infty,0)\cup(0,\infty)$</td></tr>
<tr><td>Odd function – for each x: f(-x) = -f(x)</td><td>Odd function – for each x: f(-x) = -f(x)</td></tr>
<tr><td>Decreasing on D(f) = $(-\infty,0)\cup(0,\infty)$</td><td>Increasing on D(f) = $(-\infty,0)\cup(0,\infty)$</td></tr>
<tr><td>Not bounded function on D(f) = $(-\infty,0)\cup(0,\infty)$</td><td>Not bounded function on D(f) = $(-\infty,0)\cup(0,\infty)$</td></tr>
<tr><td>No minimum, no maximum</td><td>No minimum, no maximum</td></tr>
</table>

Power **–n** is an **odd** number n = -1, -3, -5…	Power **–n** is an **even** number n = -2, -4, -6…
The graph is a *hyperbole*: 	Graph: 
Domain: D(f) = R – {0} $=(-\infty,0)\cup(0,\infty)$	Domain: D(f) = R – {0} $=(-\infty,0)\cup(0,\infty)$
Range of values: H(f) = R – {0} $=(-\infty,0)\cup(0,\infty)$	Range of values: H(f) = $R^+=(0,\infty)$
Decreasing on $(-\infty,0)\cup(0,\infty)$	Increasing on $(-\infty,0)$, decreasing on $(0,\infty)$
Odd: for each x it's true: f(-x) = - f(x)	Even: for each x it's true: f(-x) = f(x)
Not bounded	Bounded from below with 0
no maximum, no minimum	no maximum, no minimum

Ex. 1 Draw graphs of functions a) $y = x^{-4} - 1$, b) $y = \frac{1}{(1+x)^2}$, c) $y = \frac{1}{(x-1)^3}$

a) $y = x^{-4} - 1$	b) $y = \frac{1}{(1+x)^2}$	c) $y = \frac{1}{(x-1)^3}$
		

LINEAR FRACTIONAL FUNCTIONS

Linear fractional function is every function of the form: **f(x): y =** $\frac{ax+b}{cx+d}$, where a, b, c, d$\in$R; c$\neq$0; ad - bc$\neq$0.

To sketch the graph of this kind of a function it is necessary to learn the division of terms because we shall divide the numerator with the denominator. Moreover we cannot forget to set the domain, i.e. D(f) = R – {-$\frac{d}{c}$}.

Division of terms:

$(4x^3 + 5x^2 – 3x + 8) : (2x + 3) = 2x^2 – 0.5x - 0.75 + \frac{8.25}{2x+3}$

$\underline{-(4x^3 + 6x^2)}$

$(- x^2 – 3x + 8)$

$\underline{-(- x^2 - 1.5x)}$

$(- 1.5x + 8)$

$\underline{-(-1.5x – 2.25)}$

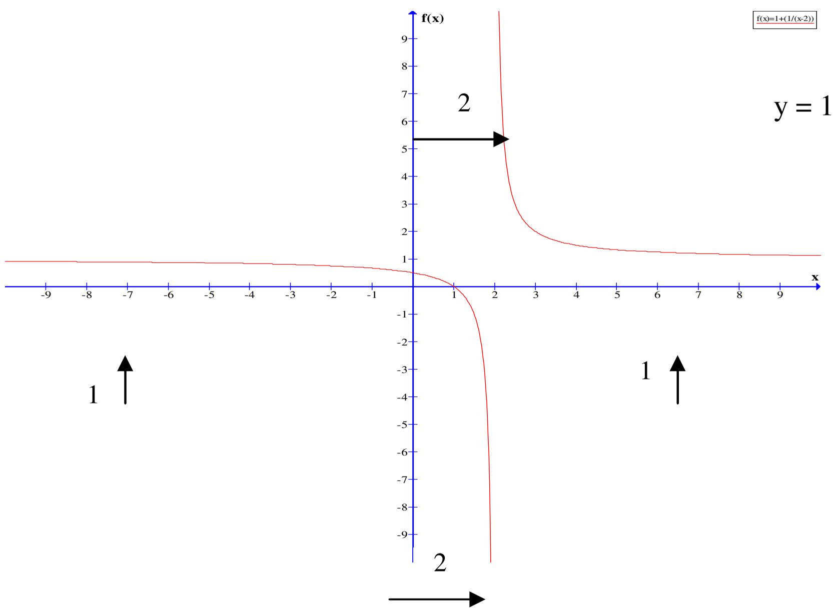
8.25

Ex. 1 Draw a graph of the function y = $\frac{x-1}{x-2}$ D(f) = R – {2}

$(x – 1) : (x – 2) = 1 + \frac{1}{x-2}$

$\underline{-(x – 2)}$

1



y = 1 + $\frac{1}{x-2}$

$$y = \pm a + \frac{k}{x \pm b}$$

+a	-a	+b	-b
↑	↓	←	→

A graph of linear fractional function is a hyperbola which is got by shifting the graph of function $y = \frac{k}{x}$ (indirect proportionality).

The number **a** out of the fraction tells us the direction along y-axis, considering its sign, the number **b,** which is out of the domain due to the denominator, tells us the direction of the shift to the right or to the left along x-axis.