

**Quadratic Inequalities**

A quadratic inequality has the form:

$$ax^2 + bx + c \geq 0 \quad ax^2 + bx + c \leq 0$$

$$ax^2 + bx + c > 0 \quad ax^2 + bx + c < 0$$

Solve the quadratic inequality  $x^2 - x - 12 < 0$

**Solution**

This inequality is solved with values of  $x$  that make  $x^2 - x - 12$  negative ( $< 0$ ).

The quantity  $x^2 - x - 12$  changes from positive to negative or from negative to positive at the points where it equals 0. For this reason, first solve the equation

$$x^2 - x - 12 = 0$$

$$x^2 - x - 12 = 0 \quad (x - 4)(x + 3) = 0 \quad x = 4 \text{ or } x = -3$$

Locating -3 and 4 on a number line, as shown in Figure 3, determines three intervals A, B, and C.  $(-\infty, -3)$ ,  $(-3, 4)$  and  $(4, \infty)$



FIGURE 3

Decide which intervals include numbers that make expression  $x^2 - x - 12$  negative by substituting any number from each interval in the polynomial. For example,

choose -4 from interval A:  $(-4)^2 - (-4) - 12 = 8 > 0$ ;

choose 0 from interval B:  $0^2 - 0 - 12 = -12 < 0$ ;

choose 5 from interval C:  $5^2 - 5 - 12 = 8 > 0$ .

Only numbers in **interval B** satisfy the given inequality, so the solution is  $(-3, 4)$ . A graph of this solution is shown in Figure 4.

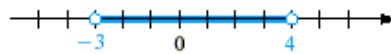


FIGURE 4

**Solve  $-x^2 + 4 < 0$ .**

I need to know where the graph crosses the  $x$ -axis. That is, I first need to find where  $-x^2 + 4$  is equal to zero:

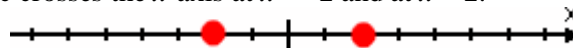
$$-x^2 + 4 = 0$$

$$x^2 - 4 = 0$$

$$(x + 2)(x - 2) = 0$$

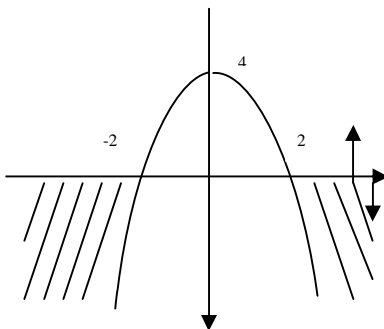
$$x = -2 \text{ or } x = 2$$

This says that the quadratic crosses the  $x$ -axis at  $x = -2$  and at  $x = 2$ .



Since this is a "negative" quadratic, it graphs as an upside-down parabola.

$a < 0$



positive part of a plane

negative part of a plane

the result is an area enclosed by a curve and  $x$ -axis  
intervals  $(-\infty, -2) \cup (2, \infty)$

Exercise

**1, Solve each inequality:**

1,  $x^2 - 8x + 15 > 0$

2,  $x^2 + 4x + 3 \leq 0$

3,  $x^2 + x - 12 < 0$

4,  $2x^2 + x - 3 < 0$

5,  $6x^2 - 7x + 2 \geq 0$

6,  $3 - 2x - x^2 < 0$

7,  $x^2 - 2x - 8 > 0$

8,  $x^2 - 2x - 15 \leq 0$

**2, Solve each inequality:**

1,  $6x^2 - 7x + 2 \geq 0$

3,  $x^2 + 4x + 3 \leq 0$

5,  $x^2 + 21 < 10x$

7,  $54 - 11x \leq 30 - x^2$

2,  $x^2 + 4x + 3 \leq 0$

4,  $x^2 + 3x \geq 10$

6,  $x^2 + 4 > 5x$

8,  $5x > 3 - x^2$

**3, Solve each inequality:**

1,  $10x^2 + 40 \geq 0$

3,  $10 - 2x^2 \leq 0$

2,  $2x^2 - 5x \leq 0$

4,  $x - x^2 \geq 0$

**4, Solve each inequality:**

1,  $\frac{2x-3}{x^2-4} < 0$

2,  $\frac{8x^2-48}{x+5} \leq 0$

3,  $\frac{3x+7}{x^2-x-30} > 0$

4,  $\frac{x^2-x-2}{x^2+4x} \geq 0$

5,  $\frac{x^2-2x-8}{36-x^2} \leq 0$

6,  $\frac{x^2-4x+4}{3x^2-5x+3} \leq 0$

**5, Solve each inequality:**

1,  $\frac{x-3}{x+5} > \frac{x-5}{x+4}$

2,  $\frac{2x+5}{2x-3} \leq \frac{4-x}{x+3}$

3,  $\frac{x-11}{x+3} < \frac{x}{x-5}$

4,  $\frac{x^2+9}{7-x} \leq \frac{x+4}{3}$