

Solving Equations with Addition and Subtraction

You can use the addition and subtraction properties of equality to solve equations. To check, substitute the solution for the variable in the original equation. If the resulting sentence is true, your solution is correct.

Addition Property of Equality	For any numbers a , b , and c , if $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	For any numbers a , b , and c , if $a = b$, then $a - c = b - c$.

Example 1: Solve $r - 6 = -11$.

$$\begin{aligned}r - 6 &= -11 \\r - 6 + 6 &= -11 + 6 \\r &= -5\end{aligned}$$

Check: $r - 6 = -11$

$$\begin{aligned}-5 - 6 &= -11 \\-11 &= -11 \quad \checkmark\end{aligned}$$

Example 2: Solve $k + 18 = -9$.

$$\begin{aligned}k + 18 &= -9 \\k + 18 - 18 &= -9 - 18 \\k &= -27\end{aligned}$$

Check: $k + 18 = -9$

$$\begin{aligned}-27 + 18 &= -9 \\-9 &= -9 \quad \checkmark\end{aligned}$$

Sometimes an equation can be solved more easily if it is rewritten first. Recall that subtracting a number is the same as adding its inverse. For example, the equation $g - (-5) = 18$ may be rewritten as $g + 5 = 18$.

Solve each equation. Then check your solution.

1. $b - 17 = -40$

2. $x + 12 = 6$

3. $z + 2 = -13$

4. $-17 = b + 4$

5. $s + (-9) = 7$

6. $v - (-12) = 10$

7. $19 + h = -4$

8. $73 = 29 - q$

9. $-3.2 = l + (-0.2)$

10. $-25 - r = \frac{4}{36}$

11. $-\frac{3}{8} + x = \frac{5}{8}$

12. $\frac{5}{9} = -y + \frac{2}{15}$

Solving Equations with Multiplication and Division

You can solve equations in which a variable has a coefficient by using the multiplication and division properties of equality.

Multiplicative Property of Equality	For any numbers a , b , and c , with $c \neq 0$, if $a = b$, then $ac = bc$.
Division Property of Equality	For any numbers a , b , and c , with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Example 1: Solve $\frac{1}{4}n = 16$.

$$\begin{aligned}\frac{1}{4}n &= 16 \\ 4\left(\frac{1}{4}n\right) &= 4(16) \\ n &= 64\end{aligned}$$

Check: $\frac{1}{4}n = 16$

$$\begin{aligned}\frac{1}{4}(64) &\stackrel{?}{=} 16 \\ 16 &= 16 \quad \checkmark\end{aligned}$$

Example 2: Solve $8n = 64$.

$$\begin{aligned}8n &= 64 \\ \frac{8n}{8} &= \frac{64}{8} \\ n &= 8\end{aligned}$$

Check: $8n = 64$

$$\begin{aligned}8(8) &\stackrel{?}{=} 64 \\ 64 &= 64 \quad \checkmark\end{aligned}$$

Solve each equation. Then check your solution.

1. $-3r = -24$

2. $8s = -64$

3. $-3t = 51$

4. $\frac{1}{4}w = -16$

5. $6x = \frac{3}{4}$

6. $1\frac{1}{4}y = -3\frac{3}{4}$

Define a variable, write an equation, and solve each problem. Then check your solution.

7. Twelve times a number is 96. What is the number?

8. One half of a number is fifteen. What is the number?

9. Negative four times a number is -112 . What is the number?

10. Regina paid \$53.50 for 5 basketball tickets. What is the cost of each ticket?

Complete.

11. If $4x = 100$, then $8x = \underline{\hspace{2cm}}$

12. If $6y = 36$, then $3y = \underline{\hspace{2cm}}$

13. If $-10a = 53$, then $-5a = \underline{\hspace{2cm}}$

14. If $2g + h = 12$, then $4g + 2h = \underline{\hspace{2cm}}$

Solving Equations with the Variable on Both Sides

When an equation contains parentheses or other grouping symbols, first use the distributive property to remove the grouping symbols. If the equation has variables on each side, use addition and subtraction property of equality to write an equivalent equation that has all the variables on one side. Then solve the equation.

Example: Solve $4(2a - 1) = -10(a - 5)$.

$$\begin{aligned}4(2a - 1) &= -10(a - 5) \\8a - 4 &= -10a + 50 && \text{Use the distributive property.} \\8a + 10a - 4 &= -10a + 10a + 50 && \text{Add } 10a \text{ to each side.} \\18a - 4 &= 50 && \text{Check:} \\18a - 4 + 4 &= 50 + 4 && \text{Add 4 to each side.} \\18a &= 54 && 4(2a - 1) = -10(a - 5) \\ \frac{18a}{18} &= \frac{54}{18} && 4(2 \cdot 3 - 1) = -10(3 - 5) \\a &= 3 && 4(6 - 1) = -10(-2) \\ & && 4(5) = -10(-2) \\ & && 20 = 20 \checkmark\end{aligned}$$

Some equations may have *no solution*, and some equations may have *every number* in their solution set. An equation that is true for every value of the variable is called an **identity**.

Solve each equation. Then check your solution.

1. $-3(x + 5) = 3(x - 1)$

2. $6 - b = 5b + 30$

3. $5y - 2y = 3y + 2$

4. $2(7 + 3t) = -t$

5. $3(a + 1) - 5 = 3a - 2$

6. $75 - 9g = 5(-4 + 2g)$

7. $1.2x + 4.3 = 2.1 - x$

8. $4.4s + 6.2 = 8.8s - 1.8$

9. $5(f + 2) = 2(3 - f)$

10. $\frac{1}{2}b + 4 = \frac{1}{8}b + 88$

11. $\frac{2}{5}w - w = -\frac{1}{5}(3w + 2)$

12. $5(p + 3) + 9 = 3(p - 2) + 6$

13. $\frac{3}{4}k - 5 = \frac{1}{4}k - 1$

14. $0.03g - (2g + 3) = 1.8$

15. $-5(2r + 3) = 3(11 - 4r) - 58$

Solve each equation. Then check your solution.

1. $4x - 9 = 7x + 12$

2. $6y - 3 = 6y + 8$

3. $8m + 13 = 13 + 8m$

4. $8n - 13 = 13 - 8n$

5. $\frac{x+3}{2} = 15$

6. $\frac{2r-3}{3} = \frac{3}{5}$

7. $1.4f + 1.1 = 8.3 - f$

8. $0.4r - 1.2 = 0.3r + 0.6$

9. $\frac{1}{2}d + \frac{3}{8} = -2d$

10. $\frac{3}{5}x - 2 = 6 + \frac{1}{4}x$

11. $\frac{5}{2}t - t = 3 + \frac{3}{2}t$

12. $4.2z = -4(0.6z - 1.2)$

13. $-3(b - 8) - 5 = 9(b + 2) + 1$

14. $8p - 5(p + 3) = (7p - 1)3$

15. $-4(2 - 3x) = 7 - 2(x - 3)$

16. $2(a - 8) + 7 = 5(a + 2) - 3a - 19$

Write an equation and solve. Then check your solution.

18. Twice a number increased by 12 is equal to 31 less than three times the number. Find the number.

19. Eight minus two times a number is equal to the number plus 17. Find the number.

20. Twice the greater of two consecutive odd integers is 13 less than three times the lesser. Find the integers.

21. The perimeter of a rectangle is 24 inches. Find the dimensions if its length is 3 inches greater than its width.