

Quadratic equation

The general form is $ax^2 + bx + c = 0$, where $a \neq 0$.

a is the coefficient of x^2 , b is the coefficient of x and c is the constant coefficient

1. Solve quadratic equations by factoring - ac method

1. $x^2 + 5x + 6 = 0$	11. $4x^2 + 3x - 7 = 0$
2. $t^2 + 5t + 4 = 0$	12. $24t^2 + 19t + 2 = 0$
3. $m^2 + 7m + 10 = 0$	13. $15t^2 + 2t - 1 = 0$
4. $k^2 + 10k + 24 = 0$	14. $16x^2 - 8x + 1 = 0$
5. $p^2 + 14p + 24 = 0$	15. $6y^2 + 33y - 63 = 0$
6. $r^2 + 9r + 18 = 0$	16. $4y^2 + 8y - 96 = 0$
7. $w^2 + 11w + 18 = 0$	17. $6t^2 + 13t + 5 = 0$
8. $x^2 + 7x + 12 = 0$	18. $3x^2 - 16x - 12 = 0$
9. $a^2 + 8a + 12 = 0$	19. $7x^2 - 37x + 10 = 0$
10. $k^2 + 10k + 21 = 0$	20. $8x^2 + 10x - 3 = 0$

2. Solve quadratic equations by completing the square.

Solve $x^2 - 6x - 1 = 0$ by completing the square. Leave your answer in the form $a \pm \sqrt{b}$

$$x^2 + 2ax = (x + a)^2 - a^2$$

$$x^2 - 6x = x^2 - 2 \cdot 3x = (x - 3)^2 - 9$$

$$\text{So } x^2 - 6x - 1 = (x - 3)^2 - 9 - 1 = (x - 3)^2 - 10$$

$$\text{When } x^2 - 6x - 1 = 0, \text{ then } (x - 3)^2 - 10 = 0 \Rightarrow (x - 3)^2 = 10$$

Taking the square root of both sides gives:

$$x - 3 = \pm \sqrt{10} \Rightarrow x_{1,2} = 3 \pm \sqrt{10}$$

1, $x^2 + 4x - 1 = 0$	11, $m^2 - 18m + 81 = 0$
2, $x^2 + 14x - 5 = 0$	12, $t^2 - 8t + 16 = 0$
3, $x^2 - 6x + 3 = 0$	13, $y^2 + 20y + 100 = 0$
4, $x^2 + 6x + 7 = 0$	14, $d^2 + 2d + 1 = 0$
5, $x^2 - 4x - 1 = 0$	15, $t^2 - 2t - 63 = 0$
6, $x^2 + 3x + 3 = 0$	16, $x^2 - 2x - 7 = 0$
7, $x^2 - 5x - 5 = 0$	17, $x^2 - 24x + 144 = 0$
8, $x^2 + x - 1 = 0$	18, $x^2 + 2x - 9 = 0$
9, $x^2 + 8x - 6 = 0$	19, $t^2 - t - 20 = 0$
10, $x^2 + 2x - 1 = 0$	20, $q^2 - q - 56 = 0$

3. Solve quadratic equations by using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity $(b^2 - 4ac)$ in the quadratic formula is known as the **discriminant**.

When $b^2 > 4ac$, $(b^2 - 4ac)$ is positive. This has been the case in almost all of the quadratics you have solved so far and it means there are two solutions.

When $b^2 = 4ac$, $(b^2 - 4ac)$ is zero. This has been the case in some of the quadratics you have solved so far. It means there is only one solution (the repeated root).

When $b^2 < 4ac$, $(b^2 - 4ac)$ is negative. So its square root is that of a negative number. Such a square root cannot be found (at GCSE level) and therefore there are no solutions.

1, $2x^2 + x - 8 = 0$	11, $4x^2 + 9x + 3 = 0$
2, $7x^2 + 12x + 2 = 0$	12, $3x^2 + 11x + 9 = 0$
3, $6x^2 + 22x + 19 = 0$	13, $x^2 - x - 10 = 0$
4, $x^2 + 3x - 6 = 0$	14, $5x^2 + 2x - 1 = 0$
5, $3x^2 - 7x + 1 = 0$	15, $3x^2 + 5x + 1 = 0$
6, $2x^2 + 11x + 4 = 0$	16, $2x^2 + 3x - 3 = 0$
7, $4x^2 + 5x - 3 = 0$	17, $5x^2 + x - 3 = 0$
8, $4x^2 - 9x + 4 = 0$	18, $2x^2 - 5x - 8 = 0$
9, $7x^2 + 3x - 2 = 0$	19, $2x^2 + 6x - 5 = 0$
10, $5x^2 - 10x + 1 = 0$	20, $2x^2 + 4x - 7 = 0$

4. Quadratic inequalities

Solve $-x^2 + 4 < 0$.

I need to know where the graph crosses the x -axis. That is, I first need to find where $-x^2 + 4$ is equal to zero:

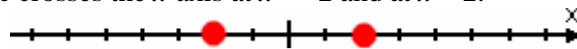
$$-x^2 + 4 = 0$$

$$x^2 - 4 = 0$$

$$(x + 2)(x - 2) = 0$$

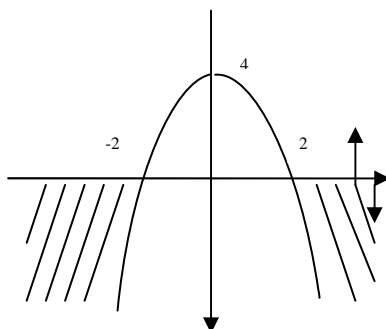
$$x = -2 \text{ or } x = 2$$

This says that the quadratic crosses the x -axis at $x = -2$ and at $x = 2$.



Since this is a "negative" quadratic, it graphs as an upside-down parabola.

$a < 0$



positive part of a plane

negative part of a plane

the result is an area enclosed by a curve and x -axis
intervals $(-\infty, -2) \cup (2, \infty)$

1, $x^2 - 8x + 15 \geq 0$	9, $x^2 + x - 12 < 0$
2, $x^2 + 4x + 3 < 0$	10, $x^2 - 2x - 15 \leq 0$
3, $x^2 + 4x + 3 \leq 0$	11, $x^2 + 4x + 3 \leq 0$
4, $x - x^2 \geq 0$	12, $54 - 11x \leq 30 - x^2$
5, $6x^2 - 7x + 2 \geq 0$	13, $x^2 + 3x \geq 10$
6, $3 - 2x - x^2 < 0$	14, $10 - 2x^2 \leq 0$
7, $x^2 - 2x - 8 > 0$	15, $2x^2 - 5x \leq 0$
8, $5x < 3 - x^2$	16, $10x^2 + 40 < 0$

5. Quadratic function, drawing a function

a) find roots

b) find a vertex V: $x = \frac{-b}{2a}$ $y = c - \frac{b^2}{4a}$

c) c – y-axis intercept

d) graph

e) properties

1, $y = x^2 + 5x + 6$	4, $y = -x^2 - 7x + 12$	7, $y = 4x^2 - 4$
2, $y = x^2 - x - 10$	5, $y = 81 - x^2$	8, $y = 5x - 10x^2$
3, $y = 4x^2 - 9x + 4$	6, $y = x^2 - 10x$	9, $y = x^2$

6. Simple interest and Compound interest

If you borrow \$2000 at 6% interest for 2 years, you have to pay the following simple interest:

$$I = P \times R \times T = 2000 \times 6\% \times 2 = 2000 \times 0.06 \times 2 = \$240$$

Meredith borrows an amount of \$10,000 from a bank and the bank charges an interest rate of 6%, compounded *quarterly*. Calculate the balance after 2 years.

Using the compound interest formula, we get,

$P = \$1000$, $R = 6\% = (6/100) = 0.06$, $n = 4$ (Remember, interest is compounded quarterly) and $t = 2$ years. (1 years = 4 quartiles)

$$A = 1000(1 + 0.06/4)^{(4)(2)}$$

$A = \$1126.492$ or $\$1126$ (approximately)

So, the amount Meredith has to pay to the bank is \$1610 and the compound interest incurred is, Compound Interest = Amount (A) – Principal (P)

$$C.I = \$1126.492 - \$1000 = \$126 \text{ (approximately)}$$

- 1, John borrows an amount of \$60,000 from a bank and the bank charges an interest rate of 4.5%, compounded *semi-annually*. Calculate the balance after 3 years.
- 2, Tom borrows an amount of \$12,000 from a bank and the bank charges an interest rate of 5.3%, compounded *quarterly*. Calculate the balance after 5 years.
- 3, David borrows an amount of \$7,000 from a bank and the bank charges an interest rate of 5.7%, compounded *monthly*. Calculate the balance after 7 years.
- 4, Meredith borrows an amount of \$5,000 from a bank and the bank charges an interest rate of 11%, compounded *annually*. Calculate the balance after 4 years.
- 5, Peter borrows \$7000 at 4.6% interest for 7 years. What is a simple interest?