TRIGONOMETRY

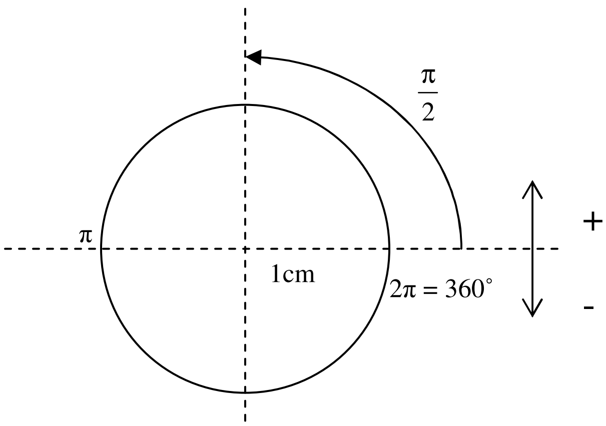
When we give the size of the angle in radians, we use the arc measure.
When we give the size of the angle in degrees, we use the grade measure.

Conversion of the angle from arc measure to grade measure $\alpha = \frac{x.180°}{\pi}$

Conversion of the angle from grade measure to arc measure $x = \frac{\pi.\alpha}{180°}$

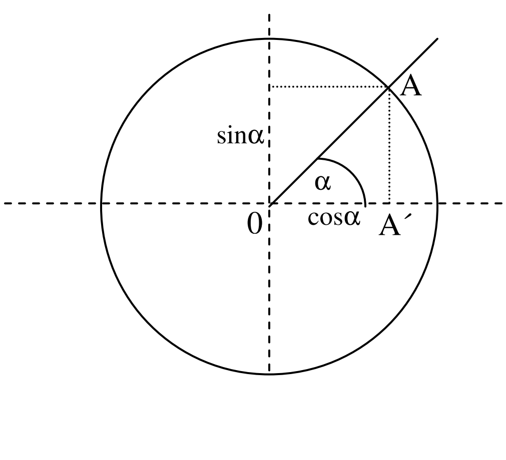
UNIT CIRCLE

Unit circle – radius r = 1cm, length of unit circle is 2πr = 2π



α	0°	30°	45°	60°	90°	180°	270°	360°
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π

The point A lies on a unit circle with coordinates



$$\cos ine\,\alpha = \frac{adjacent}{hypotenuse} = \frac{OA'}{1}$$

$$\sin\alpha = \frac{opposite}{hypotenuse} = \frac{AA'}{1}$$

Hypotenuse = radius = 1

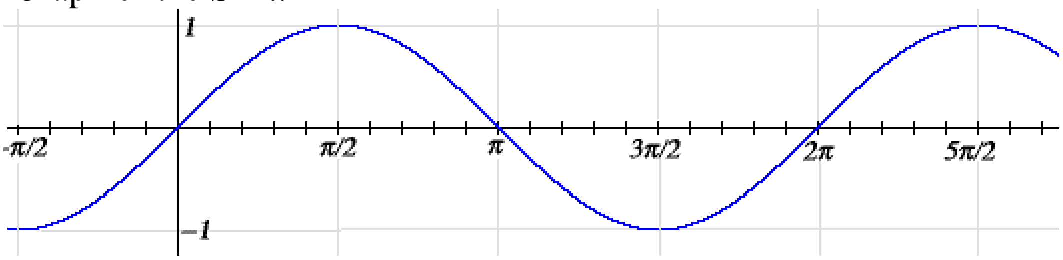
Definition:

Function sinus is a function, which binds the value y_A to each $x \in R$.

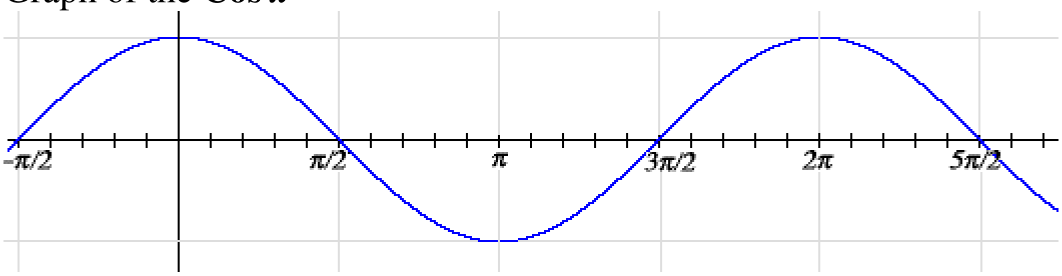
Function cosine is a function, which binds the value x_A to each $x \in R$.

PROPERTIES OF THE FUNCTIONS SINE AND COSINE

Graph of the **Sin *x***



Graph of the **Cos *x***



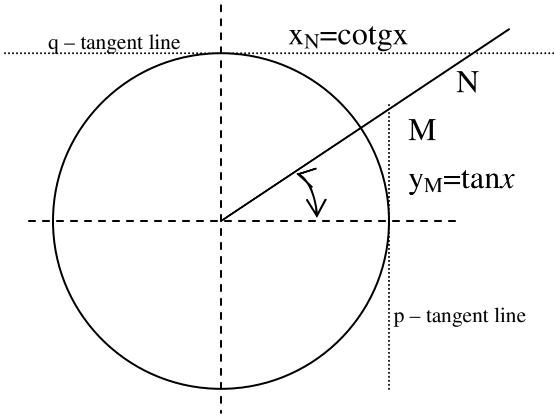
SINE	**COSINE**
Domain: D(f) = R	Domain: D(f) = R
Range: R(f) = $\langle -1,1 \rangle$	Range: R(f) = $\langle -1,1 \rangle$
Increasing on each interval $\left\langle -\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi \right\rangle$	Increasing on each interval $\langle \pi+2k\pi; 2\pi+2k\pi \rangle$
Decreasing on each interval $\left\langle \frac{\pi}{2}+2k\pi; \frac{3\pi}{2}+2k\pi \right\rangle$	Decreasing on each interval $\langle 2k\pi; \pi+2k\pi \rangle$
Odd function: sin (-x) = - sin x	Even function: cos (-x) = cos x
Bounded	Bounded
Maximum in point $\frac{\pi}{2}+2k\pi$	Maximum in point $2k\pi$
Minimum in point $\frac{3}{2}\pi + 2k\pi$	Minimum in point $\pi + 2k\pi$

Angle (x)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$y_M = \sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$x_M = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1

Quadrants	I.Q $\left\langle 0, \frac{\pi}{2} \right\rangle$	II.Q $\left\langle \frac{\pi}{2}, \pi \right\rangle$	III.Q $\left\langle \pi, \frac{3}{2}\pi \right\rangle$	IV.Q $\left\langle \frac{3}{2}\pi, 2\pi \right\rangle$
sin *x*	Incr.	Decr.	Decr	Incr.
cos *x*	Decr.	Decr.	Incr.	Incr.

Sine and cosine are periodic functions with the period **2π**.

PROPERTIES OF THE FUNCTIONS TANGENT AND COTANGENT

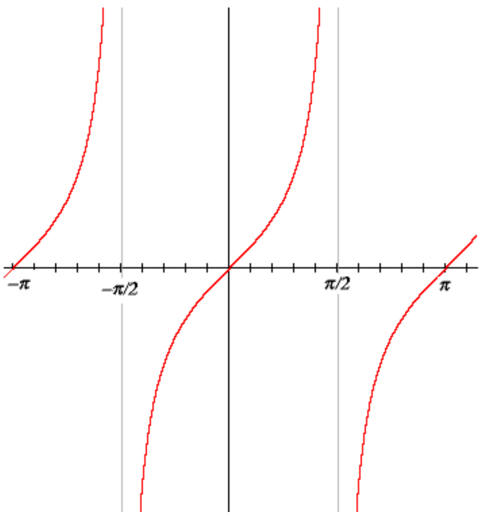


$y_M = \tan x, \quad x_N = \cot anx$

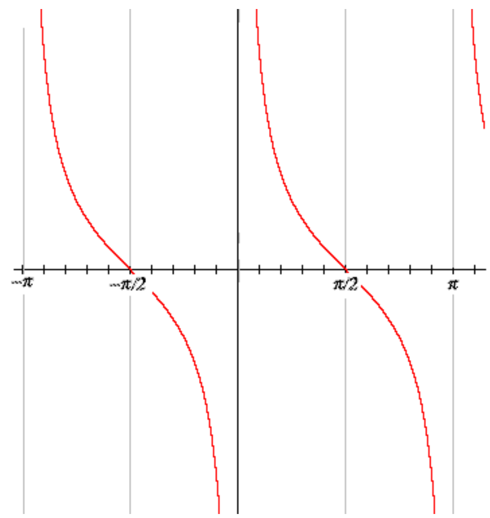
Function tangent is defined as: $y = \tan = \dfrac{\sin x}{\cos x}$.

Function cotangent is defined as: $y = \mathrm{co}\tan = \dfrac{\cos x}{\sin x}$

Graph of the **Tg *x***



Graph of the **Cotg *x***



Tangent	**Cotangent**
D(f) = R - $\left\{\frac{\pi}{2}+\pi k\right\}$; k∈ Z	D(f) = R - $\{\pi k\}$; k∈ Z
R(f) = R	R(f) = R
Increasing on interval $\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right)$	Decreasing on interval $(k\pi;\pi+k\pi)$
Odd function tg (-x) = - tg x	Odd function cotg (-x) = - cotg x
Not bounded	Not bounded
No minimum, no maximum	No minimum, no maximum

Angle (x)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	---	0	---
$\cot anx$	---	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	---	0

BASIC RELATIONS OF GONIOMETRIC FUNCTIONS

Definition: For each $x \in R$ it's valid: $\mathbf{\sin^2 x + \cos^2 x = 1}$.

For each $x \neq k \times \frac{\pi}{2}$; $k \in Z$ it's valid: $\mathbf{tg\, x \times cotg\, x = 1}$.

TRIGONOMETRIC IDENTITIES

Definition: For each two real numbers *x* and *y* it's valid:

$\mathbf{\sin(x + y) = \sin x \times \cos y + \cos x \times \sin y}$

$\mathbf{\sin(x - y) = \sin x \times \cos y - \cos x \times \sin y}$

$\mathbf{\cos(x + y) = \cos x \times \cos y - \sin x \times \sin y}$

$\mathbf{\cos(x - y) = \cos x \times \cos y + \sin x \times \sin y}$

GONIOMETRIC FUNCTIONS OF VARIABLES **2x** AND **x/2**

Definition: For each real number x is valid:

$\sin 2x = 2\sin x \times \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$.

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$,

$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}$

SUM AND DIFFERENCE OF GONIOMETRIC FUNCTIONS

Definition: For each two real numbers *x* and *y* is valid:

$\sin x + \sin y = 2 \times \sin\frac{x+y}{2} \times \cos\frac{x-y}{2}$

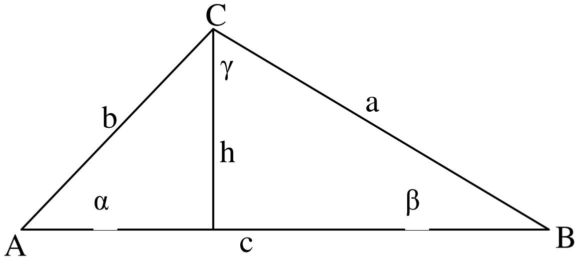
$\sin x - \sin y = 2 \times \cos\frac{x+y}{2} \times \sin\frac{x-y}{2}$

$\cos x + \cos y = 2 \times \cos\frac{x+y}{2} \times \cos\frac{x-y}{2}$

$\cos x - \cos y = -2 \times \sin\frac{x+y}{2} \times \sin\frac{x-y}{2}$

<u>GONIOMETRIC FUNCTIONS IN ATRIANGLE</u>

Definition of general triangle: a, b, c – sides of triangle; α, β, γ – angles of triangle; A, B, C – vertices of triangle; h – height of triangle



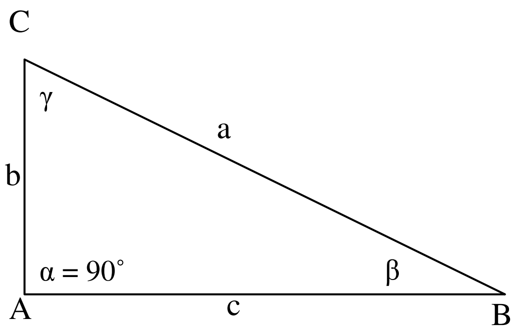
$\alpha + \beta + \gamma = 180°$

$a : b : c = \alpha : \beta : \gamma$

Area of triangle: $A = \frac{c \times h}{2}$

Length of triangle: $L = a + b + c$

Definition of right-angle triangle: one from the angles is right angle (90˚), two sides contain right angle and third side is hypotenuse.



$\alpha + \beta + \gamma = 180^\circ$ $a : b : c = \alpha : \beta : \gamma$

Area of triangle: $A = \frac{c \times b}{2}$

Length of triangle: $L = a + b + c$

a – hypotenuse

b – opposite side to angle β, adjacent side to angle γ

Sine of angle in rectangular triangle equals to relation of opposite side and hypotenuse.

Cosine of angle in rectangular triangle equals to relation of adjacent side and hypotenuse.

Tangent of angle in rectangular triangle equals to relation of opposite side and adjacent side.

Cotangent of angle in rectangular triangle equals to relation of adjacent side and opposite side.

SINE RULE

Definition: For any triangle it is valid: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

COSINE RULE AND AREA OF TRIANGLE

Definition: For any triangle ABC it is valid:

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Area of general triangle: $A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}bc\sin\alpha = \frac{1}{2}ac\sin\beta$