Relations
A relation is a set of inputs and outputs, often written as ordered pairs (input, output). We can also represent a relation as a mapping diagram or a graph. For example, the relation can be represented as:

![Mapping Diagram of Relation](image1)

**Graph of Relation**

*y is not a function of x* \( (x = 0 \text{ has multiple outputs}) \)

Functions
A function is a relation in which each input \( x \) (domain) has only one output \( y \) (range).

![Mapping Diagram of Relation](image2)

**Graph of Relation**

To check if a relation is a function, given a mapping diagram of the relation, use the following criterion:

1. *If each input has only one line connected to it, then the outputs are a function of the inputs.*
2. *The Vertical Line Tests for Graphs*

To determine whether \( y \) is a function of \( x \), given a graph of a relation, use the following criterion: if every vertical line you can draw goes through only 1 point, \( y \) is a function of \( x \). If you can draw a vertical line that goes through 2 points, \( y \) is not a function of \( x \). This is called the *vertical line test.*
In the following graphs:

- $y$ is a function of $x$ (passes vertical line test)
- $y$ is not a function of $x$ (fails vertical line test)

### Function notation

There is a special notation, that is used to represent this situation:
- if the function name is $f$, and the input name is $x$, then the unique corresponding output is called $f(x)$ (which is read as "$f$ of $x$").

We can also use letters: $g(x)$, $h(x)$ or simply $y$

#### Questions

**Question:** What does the function notation $g(7)$ represent?

**Answer:** the output from the function $g$ when the input is 7

**Question:** Suppose $f(x) = x + 2$. What is $f(3)$?

**Answer:** $f(3) = 3 + 2 = 5$ (simply substitute number 3 for the variable $x$)

**Question:** Suppose $f(x) = x + 2$. What is $f(x+5)$?

**Answer:** $f(x+5) = (x + 5) + 2 = x + 7$

### Operations with functions

Given $f(x) = 3x + 2$ and $g(x) = 4 - 5x$, find $(f + g)(x)$, $(f - g)(x)$, $(f \times g)(x)$, and $(f / g)(x)$.

- $(f + g)(x) = f(x) + g(x) = [3x + 2] + [4 - 5x] = 3x - 5x + 2 + 4 = -2x + 6$
- $(f - g)(x) = f(x) - g(x) = [3x + 2] - [4 - 5x] = 3x + 5x + 2 - 4 = 8x - 2$
- $(f \times g)(x) = [f(x)]\cdot [g(x)] = (3x + 2)(4 - 5x) = 12x + 8 - 15x^2 - 10x = -15x^2 + 2x + 8$
- \[
(f / g)(x) = \frac{f(x)}{g(x)} = \frac{3x + 2}{4 - 5x}
\]
Exercises
State the domain and range of each relation. Then determine whether each relation is a function.

1. Domain | Range
--- | ---
100 | 50
200 | 100
300 | 150

2. Domain | Range
--- | ---
3 | 1
 | 5

3. | x | y |
--- | --- | ---
1 | 2 |
2 | 4 |
3 | 6 |

Graph each relation or equation and determine the domain and range.

5. \{(2, -3), (2, 4), (2, -1)\}

6. \{(2, 6), (6, 2)\}

7. \{(-3, 4), (-2, 4), (-1, -1), (3, -1)\}

8. \(x = -2\)

Find each value if \(f(x) = 2x - 1\) and \(g(x) = 2 - x^2\).

9. \(f(0)\)
10. \(f(12)\)
11. \(g(4)\)
12. \(f(-2)\)
13. \(g(-1)\)
14. \(f(d)\)
Homework
State the domain and range of each relation. Then determine whether each relation is a function

1. Domain Range
   2 21 25 30
   8

2. Domain Range
   5 10 15 105 110

3. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

4. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Graph each relation or equation and determine the domain and range.
5. \( x = -1 \)

6. \( y = 2x - 1 \)

Find each value if \( f(x) = -5x + 2 \) and \( g(x) = -2x + 3 \).
7. \( f(3) \)
8. \( f(-4) \)
9. \( g(-1.2) \)
10. \( f(-2) \)
11. \( g(-6) \)
12. \( f(m - 2) \)

13. Use the functions below to perform the following operations:
   \( f(x) = 2x \) \( g(x) = x - 2 \) \( h(x) = x^2 \) \( k(x) = x/2 \)

   \( k(x) \times f(x) \)
   \( g(x) - h(x) \)
   \( f(x) - k(x) \)
   \( h(x) + k(x) \)
   \( f(x) \div k(x) \)
   \( g(x) \times h(x) \)